REPORT RESUMES

ED 003 005 24
TRAINING IN PROBLEM SOLVING.
BY- RIMOLDI, H.J.A. DEVANE, J.R.
LOYOLA UNIV., CHICAGO, ILL., PSYCHOMETRIC LAB.
REPORT NUMBER CRP-1015-PT. 1 PUB DATE 61
REPORT NUMBER PUB-21
EDRS PRICE MF-$0.18 HC-$4.84 121P.

DESCRIPTORS- *PROBLEM SOLVING, *PROBLEM SETS, THOUGHT PROCESSES, MEASUREMENT, MEASUREMENT INSTRUMENTS, *MEASUREMENT TECHNIQUES, METHODOLOGY, *METHODS RESEARCH, TESTING, EVALUATION, HIGH SCHOOL STUDENTS, *QUESTIONING TECHNIQUES, CHICAGO, ILLINOIS, HENMAN NELSON TEST

CHANGES WERE STUDIED IN THE PROBLEM-SOLVING BEHAVIOR OF HIGH SCHOOL STUDENTS AS A FUNCTION OF A CAREFULLY DESIGNED TRAINING PROGRAM. TRAINING WAS DEFINED AS THE DEVELOPMENT OF STUDENT AWARENESS OF PROBLEM-SOLVING STRATEGIES USED. INSTRUMENTS WERE DEVELOPED AND REFINED TO MEASURE PROBLEM-SOLVING BEHAVIOR. SPECIFICALLY TESTED WAS THE FOLLOWING HYPOTHESIS--PROBLEM-SOLVING BEHAVIOR MAY BE EVALUATED BETTER BY STUDYING THE SEQUENCE OF QUESTIONS ASKED THAN BY EVALUATING PROBLEM SOLUTIONS. SOME OF THE PROBLEMS PRESENTED SETS OF QUESTIONS, OTHER PROBLEMS GENERATED STUDENT QUESTIONS. RECORDS WERE KEPT OF THE NUMBER AND ORDER OF QUESTIONS ASKED. APPROXIMATELY 50 HIGH SCHOOL STUDENTS WITH IQ RANGES ABOVE 120 (MEASURED BY THE HENMAN-NELSON TEST) WERE PAIRED AND RANDOMLY ASSIGNED TO EITHER A TRAINING OR CONTROL GROUP. TWO IDENTICAL SETS OF PROBLEMS WERE PRESENTED TO THE GROUPS. ONE SET WAS GIVEN IN A PRETEST ENVIRONMENT. PERFORMANCES ON THE SECOND SET OF PROBLEMS, GIVEN AFTER TRAINING, WERE COMPARED WITH PRETEST RESULTS. THE SCORING METHODS USED CONSIDERED (1) NUMBER OF QUESTIONS ASKED, (2) CHARACTERIZATION OF PERFORMANCE PROCESSES, (3) VERBAL REPORTS FROM TEST SUBJECTS, AND (4) NUMBER OF CORRECT SOLUTIONS. REPORTED FINDINGS INDICATED PERFORMANCE OF THE TRAINING GROUPS WAS SIGNIFICANTLY BETTER THAN THAT OF THE CONTROL GROUP. UTILIZATION AND ORDERING OF QUESTIONS WERE ESTABLISHED FOR LATER PUBLICATION (ED 003 006). (WN)

ED003005

OFFICIAL RESEARCH FILE COPY

G[redacted] 1015
Part I

PSYCHOMETRIC LABORATORY

SPECIAL
DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE
HEW 94

TRAINING IN PROBLEM SOLVING

by

H. J. A. Rimoldi and J. R. Devane

Loyola University
Chicago, Illinois

1961

U. S. DEPARTMENT OF HEALTH, EDUCATION AND WELFARE
Office of Education
This document has been reproduced exactly as received from the person or organization originating it. Points of view or opinions stated do not necessarily represent official Office of Education position or policy.

Publication No. 21

Cooperative Research Project No. 1015

TRAINING IN PROBLEM SOLVING

by

H. J. A. Rimoldi

J. R. Devane

Loyola Psychometric Laboratory
Loyola University
Chicago, Illinois

1960 - 1961

The research reported herein was supported through the Cooperative Research Program of the Office of Education, U. S. Department of Health, Education, and Welfare.

Acknowledgements

This research was supported by the Cooperative Research Program of the Office of Education, U.S. Department of Health, Education, and Welfare. The help offered by the members of that office is gratefully acknowledged.

The completion of this research has been possible thanks to the wholehearted cooperation of Rev. John F. Sullivan, S.J., Principal of St. Ignatius High School, Chicago, Illinois. He made available to us the scholastic records of the students used as subjects and assigned space and time for the testing and training sessions.

Mrs. Eileen Meyer was responsible for the administration of the problems and the "training" of the students. Her interest in the research, her careful attention to the multiple facets that an experiment entails, and her precise recording of the interviews with the students were highly successful. Without her cooperation this research would have been impossible.

Our thanks are also extended to Miss Rosemary Zacharia and Mr. James Erdmann who helped in the testing of students and in the final analysis of the results and in the preparation of this report.

The contribution of Mr. John Haley to the development of scoring procedures has been of great value in an area where originality in the development of ideas is of basic interest.

Finally our thanks are extended to Miss Nancy Strumin and Miss Mary Jo O'Connor for their painstaking efforts in the presentation and typing of the problems and the final report.

- CONTENTS -

Chapter I - DISCUSSION OF THE PROBLEMS AND MAJOR OBJECTIVES . . . 1

Chapter II - COMMENTS ON RELATED RESEARCH 8

Chapter III - METHODS OF SCORING 12

1) Correctness of the Final Answer
2) Number of Questions
3) Utility Index
4) Characterization of Processes in Terms of Performance
5) Sequential Evaluation of Problem Solving Processes
6) Verbal Reports

Chapter IV - EXPERIMENTAL PROCEDURES AND RESULTS OBTAINED 22

1) General Considerations Concerning the Preparation of the Problems
2) Experimental Description of Sample and Problems Used
3) Comparison of Experimental and Control Groups

Chapter V - DISCUSSION OF THE FINDINGS AND CONCLUSIONS 34

Chapter VI - SUMMARY 40

REFERENCES 44

Appendix I - Problems and Instructions . . . . . . 51 pages in total

Appendix II - Tables . . . . 6 pages in total

Appendix III - Figures . . . . 7 pages in total

Appendix IV - Demonstration of Method . . . . 5 pages in total

Chapter I

DISCUSSION OF THE PROBLEM AND MAJOR OBJECTIVES

The purpose of this research was to explore: 1) individual characteristics of problem solving behavior and 2) the extent to which High School students may be trained in problem solving procedures. In the present report we shall present the work accomplished during a one year period, including the development of the instruments used for the study, the statistical methods developed to analyze problem thinking performance, and a tentative interpretation of the main findings obtained. Material assembled for the project is now being further analyzed but it will not be discussed in the present report. This concerns both methods of evaluation and interpretation of results together with a comparison of these with the more traditional approaches originally used by us in the study of processes.

In order to appraise the higher mental processes that are operative in a problem solving situation, several instruments were prepared. In Appendix I these are presented together with the instructions used for their administration.

These instruments are problems which require the subject to find a solution, whose properties are not specified beforehand. There is a goal to reach but the subject does not know "a priori" how to reach it. Sometimes the goal is vague, like specifying an area in a complex figure, other times the goal is more precise like establishing how many P's are X's. The subject does not have optional choices of possible solutions at any time during the problem solving process. Furthermore, there is no time limit in the administration of the test or restrictions placed on the number of times that the subject may want to trace back his process or request anew information previously gained.

The rationale underlying our approach to problem solving starts from certain assumptions discussed in several publications (20, 21, 22). The problem solving situation is understood as a dynamic process in which the subject tries to reach the solution of a problem by requesting that information which he considers necessary. We obtained for each subject data concerning: 1) questions asked, 2) order in which they were asked, 3) solution of the problem, 4) comments and observations gathered during the testing session. With these items we tried to interpret the problem solving process at the individual and the group levels.

Since we are not primarily interested in the correctness of the final solution, this will not be taken as the only criterion for judging the subjects' performances. It can be demonstrated that different processes may lead to the same solution. Indeed the solution itself may be understood as a final link in the chain of events that precede it. By concentrating exclusively on the solution, important individual differences may be artificially eliminated.

It is assumed that by studying the questions asked by the subject and the particular sequence that he follows, valuable knowledge can be secured in relation to each individual's approach to a problem situation. For each question asked, the experimenter provides the corresponding answer. The subject is entirely free to ask any question he wants in any order he wishes.

It could be said that two main features characterize this technique: 1) the subjects actively search, as it were, for the information, i.e., ask questions in order to obtain the information desired; and 2) the response to each question is fixed by the experimenter and is the same for all the subjects. The chain of decisions (sequence of questions asked by a subject) may be interpreted as his way of searching for those items that will provide him with information that he considers necessary to reach a solution. But, of course, according to the questions selected, there will be a corresponding set of responses which will vary for each particular subject according to his manner of approaching the problem.

In comparison with ordinary tests, this technique, rather than demanding a concrete answer to a given stimulus, requires that subjects select or generate questions to obtain the information desired. While in ordinary testing the responses to problems are usually classified as right or wrong, in the present technique neither the final solution to the problem nor the selection of a question by a subject is evaluated in these terms. Different processes will involve in most cases different sequences. It is in the study of these sequences that a description and perhaps an evaluation of each individual's process can be accomplished. Then, each individual's performance may be critically examined in terms of different hypotheses and appraised accordingly.

We are aware that though it is important to know how a group performs, one of the basic problems that has to be faced in educational matters is the individual characteristics of each student. Therefore, to be satisfied exclusively with evaluating performance in terms of averages and predefined samples may not always be the best manner of approach, nor the most logical, nor the most useful. Characterization of individual performances, beyond a verbal description, is a difficult scientific problem. Lately we have attempted to study this with promising results. Forthcoming reports will present our findings. Whether people can be classified in various categories in terms of problem solving processes is a question that still requires an answer. Also, whether problem solving ability in one type of material transfers to other material is still in many ways unknown.

As stated in our proposal, we were interested in knowing whether problem solving processes could be improved by training. Training was defined as "the development of a student's own methods of handling problems by helping him to become aware of his methods", as stated in the proposal. It was our assumption that, since we were interested in describing individual processes, types of training that may prescribe "best" methods of procedure may weaken our chances of describing how the individual really proceeds. In this respect we hoped to preserve within the experimental situation, the possibility that the individual's characteristics could emerge as freely as possible. Thus training in our context is essentially based on practice with the problems, and on remarks made by the experimenter that do not suggest or prescribe ways of handling a problem but, rather, invite the subject to analyze his own approach. The expectation is then that the subject, by himself, will eliminate certain features and incorporate new ones, in terms of his own manner of solving problems rather than in terms of a predefined method or a logical system and so forth. This approach may make possible the identification of originality in problem solving.

The questions that a subject may ask in relation to a particular problem are very numerous. Early in our research we had to face a basic decision concerning experimental methodology in the preparation of the instruments to be used in this study. This refers basically to two possible manners of approach. In one, the subject is given a set of questions that he may ask; in the other, the subject generates his own questions. It is clear that in order to satisfy the assumptions and rationale previously presented, the ideal situation should allow the subject freedom to ask any questions he wants, since by presenting a written set of questions among which he chooses the ones he wants, his spontaneity is curtailed. Our experience seems to indicate that for problems like those used in this study and others (12, 19, 22, 32) it is possible for the experimenter himself to develop all the questions that subjects typically ask. This procedure was followed in some problems. That is, subjects were presented with a set of questions written on cards. The subject was free to choose among those that he wanted in any desired order. The information corresponding to each question asked was given on the back of the corresponding card. We are confident that this manner of approach does not produce a major violation of the assumptions stated previously. In our previous experience with a large number of subjects and a variety of instruments, we have not had more than very few cases in which subjects reported that they would have liked to ask a question not presented to them. In the present research this situation arose few times.

Using this approach, the experimental situation can be better controlled and the data becomes more amenable to scientific exploration. For instance, the ambiguity of the response given by the experimenter to a question asked by the subject is controlled, since all the subjects will obtain exactly the same answer for the same question. Keeping record of the questions asked is easy.

If a subject's tactic is defined in terms of the sequence of questions asked, then for a problem with a limited number of questions all these tactics can be defined beforehand and their probability of occurrence may be established. For a problem with n possible questions the number of possible tactics will be:

$$_nP_1 + {}_nP_2 + {}_nP_3 + \ldots + {}_nP_j \ldots + {}_nP_n = \text{Total number of tactics.}$$

where $_nP_j$ indicates permutations of n things taking j at a time. The first term in this equation indicates how many sequences of only one card may be obtained, the second term, sequences of 2 cards and so on until $_nP_n$ which indicates the tactics consisting of all the cards.

It could be properly said that by writing the questions on cards that are presented to the subject we predetermine as it were, the set of questions that can be asked and that this may influence the development of the subject's process. This point has not been studied in the present research but it should be the subject matter of future investigation, especially if our interest moves towards the analysis of the creative process. But this type of study may require a thorough revision of some widely accepted psychological assumptions.

It was one aim of this research that the information gathered would allow us to describe problem solving processes rather objectively by following the subjects tactics as they developed. Mapping out the field of problem solving is not an easy matter. Some of the results obtained may help to do so, and allow for a more strict formulation of hypotheses to be tested in the future.

Changes in problem solving behavior and their relationship to other estimates of ability have been analyzed in this study. The results are presented in the following text. In the interpretation of these findings it should be considered that usual estimates of ability may not be clearly related to problem solving processes. A major difficulty in this field is setting up logically acceptable criteria in terms of which problem solving behavior can be evaluated.

Chapter II

COMMENTS ON RELATED RESEARCH

During the last few years a number of studies have discussed ways of improving teaching methods at the high school level. This has been especially true in the area of mathematics (10, 29, 30, 33). This particular aspect of research is only indirectly connected with our study, since its major concern has been the development of teaching methods and devices. It seems proper to say that the final success of any teaching program depends not exclusively on its intrinsic merits but also on the appropriateness of the program in relation to its recipients (28).

Thus, it may be advisable to complement the study of educational procedures with an interpretation of the psychological characteristics of the subjects.

The already classical studies in the general area of ability and differential ability as studied by Thurstone, Spearman, Guilford and others, start from assumptions partially different from those employed in this research. The instruments used in these studies do not in general attempt to examine processes and rely heavily on an interpretation of responses to problems.

It could be said that the field of problem solving is at the present time "poorly integrated", (9). Bloom and Broder (2) found important differences between the problem solving methods of graduate students who passed and who failed their comprehensive examinations. Further, they found that a series of remedial sessions devoted to an analysis of the subject's methods and a comparison of his methods with "model" methods led to a significant improvement in grades. Our approach to the problem differs from the one of Bloom and Broder in the experimental conditions, the training methods to be used, and the refined characterization of problem solving processes.

A technique similar to the one to be used in this study was described by Glaser, Damrin, and Gardner (11). Bryan (5) used a similar approach for the purpose of evaluating trouble shooting in electronics. The differences between their approach and the one to be employed in this study has already been discussed in several papers (20, 21).

In a study by E. R. John and H. J. A. Rimoldi (15) the sequential properties of complex reasoning were discussed in terms of performance on the PSI machine (Problem Solving and Information Apparatus). In a later study (14) by E. R. John, further results with this instrument are reported. The PSI machine may be useful in studying certain phases of highly abstract reasoning. The results obtained by Blatt using the PSI apparatus show some interesting connections between problem solving ability and certain Psychosomatic mechanisms (1).

Buswell (6) studied several factors of problem solving, employing a choice technique somewhat similar to the training instruments used in this study. In particular he attempted to define common patterns in the solution of simple problems, but instead found more diversity than similarity. He did not deal with problems of training.

A series of studies related to the present research have been published between 1958 and the present time by members of the Loyola Psychometric Laboratory (7, 8, 13, 24, 26). Most of these refer to problem solving in areas different from the one to be explored in the present study.

Bruner, Goodnow and Austin (4) examined variables that may be influential in problem solving processes. The conceptual formulation of this study is in several respects different from the one developed in the present research. Bruner (3) in a recent publication has discussed problem solving in relation to educational problems.

A recent publication by Tate, Stanier, and Harootunian (31) discussed the differences between good and poor problem solvers. These differences were established in terms of a battery of tests, mostly selected from earlier factorial studies in the area of reasoning. A test called "Thought Problems" was specially prepared for this study. A composite score, obtained by averaging scores in the tests of Verbal Reasoning and Abstract Reasoning plus scores in the Davis-Eells and in Thought Problems tests, was correlated with the Standard Progressive Matrices. The line of regression was calculated and "pupils deviating most in the positive direction from the line" are considered as having the "greatest amount of problem solving ability, as measured, independent of IQ, "while "pupils deviating most in the megative direction from the line have the least". This criterion was used to differentiate good from poor problem solvers. The major conclusions are that "good problem solvers are significantly better than the poor in nearly all tests where quality of response, accuracy or judgment is required and that, without exception, the more complex the task or the more restricted the requirements, the greater their superiority". An interesting finding is related to the fact that in the ideational fluency tests, good problem solvers are more concerned about "quality in fluency" than "fluency" itself. This seems to agree with a factor defined as "ability to work under limited restricted conditions". This factor seems to be a component of mathematical ability (17). Though the study of Tate and associates does not aim at analyzing processes in themselves it definitely aims at characterizing good and poor problem solvers. Our aim has been to describe performance in problem solving by studying the processes followed by the subjects and the possibility of introducing changes in these processes through training. Some of the variables described by Tate, Stanier and Harootunian should be analyzed in relation to our findings.

Recently D. M. Johnson (16) has found that the "processes called 'preparation' and solution", could be experimentally isolated in problem solving situations and that the performance of the subjects could be predicted in terms of the training series that preceeded the crucial testing of the hypothesis.

Chapter III

METHODS OF SCORING

Some of the scoring methods used in this study have already been published. Other approaches were developed as part of this project (25, 23) and tried on a limited sample of the data. The results will be published in the near future (25).

1) Correctness of the Final Answer. For every subject taking the test a record was made as to whether or not he reached a correct solution. Our rationale implies that this may not be the only or the best criterion to judge performance since correct answers may be the result of different types of processes. The same rationale applies to incorrect responses. Table 2 shows the number of correct answers for each one of the problems used in this study.

2) Number of Questions. This score is obtained by counting the number of questions asked by each subject before reaching the solution to the problem. The properties of this score have been analyzed in (24).

Comparisons were made between the experimental and control groups and between some of the tests. The results of these observations are given in Tables 2, 3, and 5 (Appendix II).

In interpreting this score the assumption that the fewer the number of questions asked the better the problem solving ability of the subject should be avoided in many cases. In certain problems the "best" problem solvers may ask for more questions than subjects with less ability. Being aware of the intrinsic difficulties of a problem may demand a more thorough exploration.

3) Utility Index. It may be postulated that a question asked by a large number of persons in a group is more useful than one selected by few members of the group. The utility index is an empirically derived property of the question and it is defined as the ratio between the frequency with which it has been selected and the total number of subjects in the group, i.e., $i = \frac{n-1}{N-1}$, where n is the number of subjects in the group who employ the question and N is the number of subjects in the group. A more detailed explanation is given in (26). It does not consider the order in which the question was chosen. It has been shown (24) that the utility index of a question varies in relation to the group used for its computation. Levels of training and ability of the members of the group will be reflected in the utility indexes.

Comparisons between utility indexes obtained in different groups has been performed in other studies and have been used to characterize groups (26, 24). In the present research the utility indexes of the cards have been computed for the different groups of subjects and have been used to develop more elaborate scoring procedures.

Scores like utility score, corrected utility score, median value of a question have not been used in this research (20).

4) Characterization of Processes in Terms of Performance. By accumulating the utility indexes of the questions from high to low a maximum curve can be obtained. This assumes that a subject who wishes to maximize his information at every step in the process may order his selection of questions in terms of decreasing utility indexes. The minimum curve is obtained by accumulating the utility indexes in the reverse

order. In terms of the previous assumptions this would represent the "worse" possible performance.

By plotting the maximum and minimum curves an area will be enclosed. Within this area will be located the performance curve of any individual scored in terms of the utility indexes used to compute the maximum and minimum curves. If the utility indexes of all the cards are the same, then the two curves will collapse into a single straight line with a slope proportional to the constant utility index. Assuming that maximum discrimination is obtained when the questions have only utility indexes of 1.00 and .00, then the maximum and minimum curves will delimit a parallelogram. In most tests some questions will have high utility indexes and some low utility indexes. In this case by plotting the maximum and minimum curves ellipsoids will be obtained, as shown in Figures 1 through 7 (Appendix III). The slope of the maximum and the minimum curves and the area of the ellipsoid can be used advantageously for the purposes of scoring. Complete details about this procedure are given in several published studies (13, 27).

This approach can be used to study both the characteristics of groups and the properties of tests. In the first case the same test can be administered to different groups. In the second case the same group will be administered different tests. Inside the area enclosed by the maximum and minimum curves individual performances may be graphed and studied in terms of slope of the curve at different moments during the process, plateaux, final point reached, etc. Also at every step in the process a comparison may be made in terms of the distance between the individual performance curve and the maximum and minimum curves. These individual performance curves are obtained by accumulating the utility indexes of the questions selected by the subject in the order he selected them.

Let us assume that maximum and minimum curves, for a given problem are those presented in Figure 1, Problem 7 (Appendix III). Notice that the number of cards for the Test is 16 and that the cumulative sum of the utility indexes is 5.20. If the discriminatory property of the questions would have been perfect then we would have only questions with utility indexes of 1.00 and .00 and, in this particular case, to reach the value corresponding to the total sum of the utility indexes, 5.20 questions would have been needed. In this manner the parallelogram presented in this Figure has been built. The more the group agrees about questions that should be asked and not asked, then the more the maximum curve will approach the top and left edges, and the minimum curve the low and right edges of the parallelogram. The ratio between the areas enclosed by the parallelogram and the area enclosed by the maximum and minimum curves is an indication of the agreement between the members of the group concerning the usefulness (utility index) of the questions. For instance it may be postulated that training may bring greater agreement among subjects as to which questions are useful and which are not useful in the solution of a problem. In this case the value of the ratio will be high by comparison with the ratio obtained from a group of subjects where agreement is less. The ratio will be 1.00 if there is perfect agreement and .00 is there is no agreement. In the latter case it may be hypothesized that all questions are chosen with approximately the same frequency because they are interpreted by the subjects as having the same usefulness.

For a clear interpretation of this score a considerable amount of investigation is still required. Nevertheless some points may be raised to understand better the inherent limitations. We are postulating an ideal type of performance defined by the maximum curve under the assumption that subjects proceed by maximizing the information at every step in the process. The only criterion for order is the magnitude of the utility indexes. But it happens that questions are not necessarily selected in an order dictated only by their usefulness. That is, usefulness is not only a question of relative frequency of selection but also depends on proper timing. As a matter of fact questions with high utility indexes may be chosen at the end of the process, and it is conceivable that good problem solvers may want to restrain from asking a very useful question until others, that are not considered so useful, have been selected. Then his tactic may be at variance with those of the rest of the group or with the assumptions previously stated. When other psychological factors are considered, among which subjects' own stereotypes are very important, it becomes clear that the characterization of processes as here described represents one way of studying performance under a limited hypothesis.

When using this type of score great caution should be used in the interpretation of the results. The individual performance curves of the "best" problem solvers may not necessarily be those that are nearer to the maximum curve nor those that show a steeper slope.

Notice that in defining the maximum curve order is a function of decreasing utility indexes while on the contrary, when describing each subject's performance the utility indexes are accumulated as they occur in his sequence.

5) Sequential Evaluation of Problem Solving Processes. Some of the objections previously raised are contemplated in a scoring system that is now being experimentally tried. Though it does not solve all the problems that may be raised it provides means of exploring aspects of the problem solving situation not yet considered in our previous methods. In the present report we shall present it using an arbitrary example (Appendix IV).

A complete description of the method is now being printed (25) and a preliminary report that presents the main assumptions has already been published (23).

The method is based on the assumption that a more complete evaluation of problem solving performance should consider both questions asked and not asked as well as order in which questions are asked. The values for the questions asked are equivalent to utility indexes as previously described with the qualification that a question will have different utility indexes according to its order of selection. For a particular problem a set of values can be obtained either by administering the problem to a group of people and experimentally determining these values or, tables can be prepared in which the values will represent the frequency of choice of a question at a given order as postulated by the hypothesis under consideration. In all cases matrices of values based on actual observations or on different hypotheses can be prepared.

The performance of any subject can be represented by a curve in which the successive steps are given by accumulating the value of the questions asked in the order in which they were asked. These performance curves show some interesting features, for instance the fact that for the same questions asked the shape of the curve will vary according to the order of selection. These curves may be prepared using the matrix of values obtained either by observation or in terms of certain hypotheses, and thus each individual's performance may be analyzed with references to several assumptions. The sum total of the values corresponding to each performance is easily obtained and can be interpreted as a summary statement of the tactic followed by the subject. Since there is a limited number of questions that may be asked there is also a limited number of tactics and their sum total may be predetermined. The sum total of questions not asked can be also obtained and represents, as it were, the complement necessary to characterize performance more fully.

Let us assume that we decide to use the values of the matrix obtained experimentally to score a sample of subjects. These values will give, when cumulated in their proper order, the observed performance curve and the sum total for each subject. By subtracting from each cell of the matrix of observed values the corresponding values expected in terms of a hypothesis it is clear that the difference will be zero whenever the observed and the expected value are the same. Any departure from zero, either positive or negative, indicates discrepancy between observed and expected performance. Levels of significance for these differences may be computed using chi square or other appropriate statistical approaches. Work on this aspect of the problem has not yet been completed.

By cumulating the differences between the observed and expected values individual curves may be drawn. These curves indicate departure from the hypothesis at each step in the process. The interpretation of these difference curves becomes more meaningful when compared with the curves of observed performance. Again the sum total of these differences is a summary statement concerning each tactic in terms of certain hypothesis. Similarly for questions not asked.

The sum total for questions asked can be plotted against the sum total of the differences computed under any hypothesis. This graph will consist of a set of points, each one of them representing a tactic. The points can be interpreted as belonging to a polygonal convex set. Similarly for questions not asked. When the same observations are compared with other hypotheses different convex sets are obtained, since the location of the points corresponding to the observed minus expected performance will be, in most cases, different.

Some intriguing findings have been observed using this approach:

a) some tactics show great variation when examined under different hypotheses while others remain more stable,

b) there seems to be a relationship that is not yet clear between families of tactics and boundaries of the convex sets,

c) the number of sides limiting the convex sets varies with the hypothesis under study,

d) the convex sets for questions not asked show that tactics that are close together in terms of observed performance may be wide apart in terms of questions not asked,

e) the boundaries of the convex sets show certain maximum and minimum points that may be interpreted as referring to tactics that maximize or minimize information or lack of information.

In the present research two main hypotheses have been tested in relation to one of the problems. The results will be published in the future.

The distribution that could be expected in terms of combinational theory between frequency of choice and order of selection of a question and length of a sequence has been established and will be published shortly.

Though it is true that the method described and illustrated in Appendix IV is based on group performance it allows for a considerable number of comparisons in terms of a variety of hypotheses. As it were, each tactic can be cross-examined in relation to different assumptions. Individual performance curves in the data thus far examined show some interesting features. The interpretation of these in relation to the problems themselves is illuminating and it is our hope that it may help to study individuals more thoroughly and to detect feature characteristics - at the subject level - of originality in thinking and creation. In this area much work is still to be done.

6) Verbal Reports. For each subject a record was made of his verbal comments, both during the performance of the test and in the "counseling" session that followed. These findings are not presented in this report but will be the subject matter of a Master's dissertation being prepared by one of the co-workers in the project.

Chapter 4

EXPERIMENTAL PROCEDURES AND RESULTS OBTAINED

1) General Considerations Concerning the Preparation of the Problems. In the development of the tests specially prepared for this research several considerations were taken into account.

In order to meet the prescribed schedule, and at the same time have the subjects solve a substantial number of problems it was necessary to prepare instruments that could be solved within a reasonable time limit (20 to 30 minutes). This increased the difficulty of the test construction task considerably.

Our experience with instruments as those prepared for this research was limited and we had to be guided by experience as results were forthcoming. This referred to presentation of the problems, content of the problems and "face" difficulty of the problems. The verbal reports and the observations of the psychologist administering the test were valuable guides in directing our preparation of new instruments. Many of the problems used were previously administered to college students and accordingly dropped or incorporated in our final battery. But this could only be of limited help because of the differences in scholastic achievement and age, among others, between the experimental and pre-experimental samples. Also problems clearly related to academic subjects, such as mathematics and physics were developed but were not included in the research proper. Our policy could have been to construct "a priori" a set of instruments and use them for testing purposes. But the "neatness" of the design might have been hampered by inappropriate problems.

Furthermore the risk of introducing in the instruments only the preconceived ideas of the experimenter would have been much greater and our freedom in exploring promising clues might have been restricted. Also in this research we tried to avoid influencing subjects by prescribing "best" ways of solving problems in the hope that the subjects themselves would modify their approach as training proceeded. If such a result could be obtained and if it can be demonstrated that through experience subjects develop similar methods through the development of their own procedures then experimental control of this phenomenon may be achieved.

The basic principles that guided us in the preparation of the instruments can be inferred by analyzing them (Appendix I).

In the text that follows their crucial features are summarily presented.

2) Experimental Description of Sample and Problems Used. One-hundred fifty students, whose I.Q.'s were above 120, as measured by the Henmon-Nelson Test of Mental Ability, were selected from the Freshman class at St. Ignatius High School. A letter was sent to the parents of the students, outlining the character and purpose of the research, and requesting them to check "Yes" or "No" on an enclosed postal card. One hundred and five responses were received; of these, 104 were checked "Yes".

Twenty-five pairs of subjects were assembled, such that the members of each pair had identical or similar I.Q. scores and were in the same class section. Assignment of the students to class sections is based on general academic achievement. One subject was randomly selected from each pair to serve as experimental subject. The means and standard deviations of the I.Q. scores of the two groups so formed are given in Table 1.*

Several problems arranged in the format described above were investigated in a preliminary study with college students. Two of these problems were selected for administration to both the experimental and control groups. The first of these (Problem 1, Appendix I) was chosen chiefly as an illustration of the subject's task. The second (Problem 2) was judged to be of moderate difficulty, solvable in terms of several obvious procedures. In Problems 1 and 2, the subject is presented with a figure containing a number of labeled areas. He is instructed to discover a pre-selected area, using the questions presented. A record was kept both of the order in which each subject selected questions and of the solution he reached. The performances of the two groups on these problems are presented below. These differences are compared to the post-training differences.

Each member of the experimental group met individually with an investigator every two weeks for a forty-minute training session. The subject was presented with a problem and a number of questions he might employ in working towards a solution. A record was kept of the order in which he chose questions. When he felt he had arrived at a solution, he and the investigator reviewed his performance step by step, while the subject reconsidered his successive decisions. The investigator encouraged the subject to question his own thinking, and to propose alternate methods that he might have employed. The investigator was careful not to evaluate the solution offered. The only criticisms or revisions were those of the subject. The subject's reasoning as he proceeded through the problem and his questioning of his own thinking after reaching a solution were recorded in order to relate his thinking to more objective measures. Work on this area is now in process. A subject worked through two or more such problems in each training session.

---

* All tables are presented in Appendix II.

Each experimental subject participated in seven training sessions during the second half of the academic year 1960-1961. The sessions were two weeks apart. A total of twenty-two experimental problems (Problems 3 through 24) were prepared as training instruments. Because of individual differences in the rate of working the problems some subjects were presented with only 18 problems in the seven training sessions.

The preliminary study of college students had made it clear that the interest of the subjects would not be sustained if the same type of problem were employed throughout the study. A variety of problems was therefore developed, both to maintain a level of novelty, and to obtain data on a range of problems that would be of use in later, more refined continuation of the research.

Table 2 presents, for each problem, the number of students attempting it, the number of students solving it correctly prior to the review phase of the problem, and the mean and the standard deviation of the number of cards employed in its solution.

Session I. Problems 3 and 4.

Problem 3 was a variant of the type represented by Problems 1 and 2. It was constructed in an attempt to gauge the extent to which familiar dimensions of a problem will be employed, even when these do not lead to the most economical or elegant solution. The majority of subjects preferred to seek the pre-selected area by a successive narrowing down of its location in the figure. Even in the review phase, few subjects discovered that other and more subtle features of the areas, such as the number of sides, the number of geometric figures in which it lay, etc., provided complete solutions. Because of certain features of this problem, performance in it was analyzed using the method developed in this research. The results will appear in a forthcoming report.

Problem 4 requires that a subject integrate several converging lines of evidence, each of which is indeterminate by itself. Taken together, however, the set of possible solutions is quite limited. The subject was required to furnish the correct word for each blank cell in the problem that has a question mark. It was intended that each word in Column A after the first should be a synonym of the word immediately above, under a varying aspect. It is clear that this was only approximately realized. Column B provided a rhyme for each word in Column A, while Column C was intended to indicate the number of letters in the corresponding word in Column A. It will be noted that all four-letter words in Column A correspond to 'gone' in Column C, and all five-letter words to 'await'. These clues were to be discovered by the subject. This problem was not successful, insofar as it did not elicit planned approaches, nor did the review reveal much more than the frustration of the subjects. More recent versions of the problem, employing the convergence of several series, were not tried with the present sample of subjects.

Session II. Problems 5 and 6.

Problem 5, in which the subject is required to discover an open path between two points, was solved by most subjects only in the review phase of the experimental session. They typically accepted the starting point given by the directions, and would become lost in a cul de sac, returning, as indicated by the questions asked, an arbitrary number of steps toward the starting point before seeking a new path. In the review, however, most subjects determined the last steps that actually led to the goal, only then studying the beginning and intermediate steps in terms of their usefulness in leading to one of the open final paths.

Problem 6 requires the subject to determine the relationships between two formal sets by means of their mutual relationships to other sets. The subjects seemed to find the task mysterious, at first, but soon grasped what was to be done. Many subjects found this task very interesting; others clearly disliked it. The latter, in general, did not profit from the review of their procedures.

Session III. Problems 7 through 11.

This series of problems resembled those employed in traditional studies of concept formation, in that the subject is required to label an area of the figure, employing principles to be discovered in the labeling of other areas. It was intended that the problems within the series would be of increasing complexity, proceeding from a two-category system of classification to a three-category system, and from a one-feature definition of a category to a two-feature definition. The increasing difficulty is reflected in the increasing mean number of cards in Table 2.

Table 3 presents the correlations between the number of cards employed in the different problems of the same basic type within a session, for those problems completed by 15 or more subjects. The correlations for Session III indicate that there was little or no consistency in number of cards from problem to problem. Further, Figure 1*, which presents the ellipsoids, as described above, for the first three problems in the Session, indicate that Problem 7 elicited a relatively narrow range of procedures, while Problems 8 and 9 elicited decreasing agreement regarding the profitable approaches to be followed. The data do not permit a distinction between the differential effects of set from problem to problem and actual differences between the problems.

Session IV. Problems 12, 13, 14, 15, 16.

Each problem in this series presented the subjects with a task identical to those of Session III. The problems involve, presumably, successively more subtle clues throughout the series. The mean number of cards (Table 2), and the correlations between problems (Table 3) suggest that the differences introduced in this series were less drastic than those introduced in Session III. Further, the ellipsoids of Figure 2 show that there was considerably greater agreement in Problems 13 and 14 than in Problem 12. The amount of agreement did not systematically alter throughout the series.

* All figures are presented in Appendix III.

Session V. Problems 17, 18, 19.

Problem 17 was designed to require the subjects to employ clues of a spatial or geographic type, without regard to the features of the individual areas. Problem 18 required the subjects to employ more descriptive features of the areas. The mean number of cards, the correlation between the number of cards in the two problems, and the ellipsoids of Figure 3 do not indicate that the changes introduced in Problem 18 disorganized the subjects' approaches. If the changes introduced in Problem 18 had led to confusion then it could be expected that the mean number of cards would have increased, the correlation between number of cards would have been low and the ellipsoid for Problem 18 should have yielded a lower ratio.

Problem 19 is similar to Problem 6, described above, except that familiar terms are employed instead of symbols. Problem 19 does not require the subject to ignore his intuitions and experience regarding the relationships used in the problem. The relationships involved parallel those of common sense. It was perhaps for this reason that the problem did not seem to involve much difficulty for the subject.

Session VI. Problems 20, 21, 22.

Problem 20 is a simple version of the set type of problem, employing familiar terms rather than formal symbols. Problem 21 requires the subject to discover a pre-selected area, as do Problems 1, 2, 3, and 25. However, no simple locational information, such as the subjects typically look for, is made available, and all the questions are answered in the negative. As a result of both factors, relatively few of the subjects obtained the correct solution, and a relatively large mean number of questions was employed (Table 2). The absence of clear locational questions apparently made the subjects' record keeping more difficult, while the negative responses forced the subject to revise his hypotheses after each question.

Problem 22 requires the subject to solve a three-step distance-time problem, employing different rates for each step. He must employ unfamiliar measurement units. Most subjects explored the new system in order to translate the units into miles and hours, realizing only in the review phase that the translation was unnecessary, and that time-distance-rate relationships were quite sufficient for the solution of the problem.

Session VII. Problems 23, 24.

Problem 23 is a set problem using familiar terms. Problem 24, also a set problem, uses arbitrary symbols. The problems are similar in that each familiar term in Problem 23 has a corresponding symbol in Problem 24. Each question and answer in the two problems match exactly, *save for the names of the things related.* Problem 23, it might be added, does not require the subject to accept a nonsensical or conventionally implausible relationship for the purposes of solving the problem. Table 2 indicates that the problem in symbolic form, Problem 24, required more questions than did Problem 23. The correlation between the number of cards employed (Table 3) is not significant.

The ellipsoids, Figure 4, indicate considerably greater agreement in Problem 23 than in Problem 24. It appears that the familiar, concrete terms provided a powerful orienting influence, as well as making the processing of information more economical.

3) Comparison of Experimental and Control Groups. The Experimental Control comparisons were performed regarding the academic grades of the subjects and regarding the pre-training and the post-training problems.

The experimental phase of the study covered the second half of the academic year 1960-1961. In employing grades as criterion, then, the midyear grades constitute the pre-experimental level, and the appropriate test is in the difference between mean changes, based on matched pairs (18). The results are presented in Table 4. It will be observed that all the values declined from the midyear to the final grades. The reasons why grades dropped we can not explain, but our test is concerned with differences between experimental and control changes in grades. The changes in grades assigned by the teacher are of interest rather than their absolute value. The differences in changes is positive for all the 3 academic subjects judged to be of interest here as well as for the overall average, although only the difference in change in Mathematics grades is significant at the .05 level using a two-tailed test of significance. It is by no means certain that effective training in problem solving would be reflected in academic achievement; similarly, in this study, improvement in academic achievement does not permit a hard and fast judgment regarding the effectiveness of the training in the improvement of problem solving. Nevertheless, a qualifi conclusion that the training was related to gain in mathematics grades is in order.

Tables 5 and 6 present a summary of the numbers of questions employed and the number obtaining the correct solution in both groups on the pre- and post-training problems. The number of questions employed in the solution of a problem is one way of evaluating the method followed. Other factors being equal, the more efficient procedure is to be judged superior to the less efficient. However, there are many other features to a method, which, from the standpoint of this research, are of greater interest. The number of questions employed is here offered as an imperfect standard for the evaluation of a performance.

Problems 1 and 2 administered in the pre-training session yielded only minor differences between the groups in the number of questions employed. In the post-training session, the Experimental subjects employed significantly fewer questions than the Control subjects in Problem 25. The groups had identical means in Problem 26. There were no significant differences between groups in the number of correct solutions either to the pre-training or the post-training problems.

Figure 5 presents the ellipsoid generated by performance of the Experimental group and that generated by the performance of the Control group on Problem 2. The value of R is the ratio of the area of the ellipsoid to the area of the parallelogram as described in the chapter on Methods of Scoring. These values, .602 and .597, indicate that the groups were approximately equal in degree of homogeneity of approach. It can be said that the problem had the same degree of structure for both groups. Since Problem 1 was intended primarily as a demonstration problem, the ellipsoids will not be presented.

The ellipsoids for the two groups for Problem 25 are presented in Figure 6. The values of R for the Experimental and Control groups, .794 and .489, indicate that the members of the former group were, relatively, of one mind regarding the critical features of the problem; the members of the latter group employed relatively diverse approaches.

Figure 6 presents the ellipsoids generated by the performances of the two groups on Problem 26. The values of R, .621 and .551, suggest that there was slightly greater agreement among the Experimental subjects than there was among the Control subjects.

The increased agreement evident here is taken as a reflection of an increased capacity to grasp the salient features of the problem, to delineate its outlines prior to solution, as opposed to a relatively unplanned and less clear delineation. Other interpretations are valid, of course, and the one preferred here will only be established by further research.

Chapter 5

DISCUSSION OF THE FINDINGS AND CONCLUSIONS

Attempts were made to construct problems that would present various facets of the problem solving process. At the beginning of this research, it was necessary to determine whether or not certain effects could be produced. Following upon the results of this step, the formulation of adequate theory and the experimental manipulations of the effects became possible.

It is clear that, other things being equal, subjects organize a problem around its most striking features, and employ the subtle features of the problem only when necessary. Many of the problems presented the same apparent structure to most of the subjects, and suggested one general plan for their solution. For example, in Problems 1, 2, 3, 21, and 25, the subjects typically began with questions regarding the location of the area sought, and would press this approach even after its usefulness had been exhausted. Questions that classified the areas into non-contiguous groups were avoided as long as it was possible to do so. Quite often, when the subject had yet to distinguish among a limited number of adjacent areas, he would use questions that apparently defined the areas into groups, but in the light of the information he possesses, were equivalent to proposing one area after another as the solution. Such a procedure, which appears on the surface to be hypothesis testing, is actually the same as asking, "Is it J? Is it K?" and so on through the remaining possibilities.

Similarly in all the problems of Session III and IV, and the first two problems in V, the subjects typically looked first for a pattern occurring horizontally, then looked for one occurring vertically. Many subjects were quite unwilling to classify on some other basis, even though many obvious possibilities were present, and would either impose some semblance of a pattern of the labels, or would ask questions without plan, hoping, it appears, that the solution would become manifest. The review phase of the training sessions apparently was of use in this regard, a judgmental statement that must await further research for rigorous experimental support.

Problem 5 was constructed as another situation requiring the subject to re-organize his planning at several steps, with the addition that the need to give up apparent progress would be more explicit than in other problems. The problem was successful in this effort. It was observed, curiously enough, that many subjects were much more discouraged by an obstacle appearing late in the problem than by an obstacle appearing earlier. The judgment of the extent of the gains that must be forfeited, i.e., apparent progress toward the goal that must be abandoned, appeared a difficult one to make. Some subjects in effect gave up the problem while others would return to the origin and begin again. The capacity to decide just how much needs to be abandoned is worth attention and will be studied further in the next phase of this research.

The requirement that the subject employ information presented in a negative form was built into several problems. As several previous research studies have shown, such information is not as usable, in general, as information expressed positively. However, the effect does not hold under all conditions. It appears that information is less useful when it disrupts a train of thought; subjects tend to phrase their hunches positively, and the negative statement is one that compels the subject to give up what he has come to regard as possibly true. In Problems 6, 19, 20, 23, and 26, for example, the expression of information in negative form did not seem to add to the difficulty of the problem. The effect of this variable in Problem 21 cannot be observed.

In Problem 4, the subject was required to discover the principles that were operating, just as he was required to do in other problems. However, he was then required to use the principles in producing the correct response, rather than to use the principles in further classifying. The rules governing the exemplars were here too difficult to detect. The rules are more obvious, though not more determinate, in more recent versions, and the requirement that the subject create his own instances will be used in the future.

Sessions III, IV and V were designed to determine, among other things, the extent to which the solution to one problem would influence the approach to subsequent problems of the same type in the same session. By changing some features of the problems it was possible to observe whether or not this expectation effect could be reduced. The extent of the effect was very great in the earlier problems, almost to the point of being disabling. This statement is bolstered by the number of cards employed in adjacent problems in Session III. It appears, from the values obtained in Sessions IV and V, that the subjects learned not to be misled by previously successful procedures. The subjects' expectations regarding the range of possible solutions seems to depend on the range of solutions previously encountered. If the level of complexity has varied in previous problems, the subjects do not appear to commit themselves so thoroughly to a limited number of possibilities on subsequent problems.

The performances of the subjects on these same problems also illustrated persistent individual differences in the capacity or the willingness to submit theory to rigorous test. Some subjects were consistently satisfied with relatively small amounts of supporting evidence, and would use it as the basis for the solution offered. The critical testing of a conviction appeared to involve too much risk. A few of the subjects, on the other hand, consistently sought out evidence that would, if true, destroy their theories. It cannot be clearly determined from the data whether the former subjects became more critical of their own work as a result of discussing their efforts with the investigator, but, on a judgmental basis, this does seem to be true.

The greater agreement in Problem 23 over that of Problem 24 indicates that the use of familiar terms gave a structure and a plan to the former problem. The greater difficulty of Problem 24, as reflected in the number of questions employed, is not so readily interpreted, since the two problems had precisely the same logical complexity. Perhaps the familiar terms provide a sort of mnemonic device, making the handling of relationships less taxing. Research currently in progress will examine the differences between performances in similar problems, with variations in the plausibility of the relationships given in the problem using familiar terms. In some of these, the subject will be required to ignore his intuitive grasp of real-life, commonsense relationships.

The statistical findings have been presented in greater detail in previous sections of this report.

It is interesting that the experimental group showed an improvement over the control group in grades in mathematics. This improvement was significant at the .05 level when comparing differences in mean changes. For all the other academic subjects considered in this research the trend was the same though the obtained values showed no significance.

Of these academic subjects it seems that the only one that requires problem solving in a "formal" way is mathematics. If this interpretation is accepted then it may be postulated that training in problem solving produces an effect that is related to achievement in mathematics. We would like to avoid a firm conclusion that would link this improvement in mathematic grades to training in problem solving, but this is certainly worth exploring. We plan to study what occurs with other academic subjects that involve more problem solving activity than English or Latin, - such as Physics, Natural Science, etc.

In terms of the ratios of the ellipsoids for both experimental and control groups in problems 25 and 26 we find that there is in both problems a greater agreement for the experimental subjects than for the control subjects. When inspecting the number of questions asked it is found that in Problem 26 there is no difference between the control and experimental groups, while in Problem 25 the latter asked for less questions (see Table 5, Appendix II).

It may be proper to say that training has increased the degree of agreement between subjects and that this agreement refers to the expected utility of the questions as reflected by the utility indexes rather than to number of questions. As far as our experience indicates it seems that training tends to produce agreement in the subject's problem solving processes.

Notice that Problem 25 belongs to the general type of problems that were predominantly used in this research. This may partially explain the greater agreement of the experimental subjects in Problem 25 over Problem 26.

Finally in the interpretation of our findings it should be considered that all the subjects used in this research had an I.Q., as estimated by the Henman-Nelson Test of 120 or higher. This introduces a selection effect that in all likelihood will tend to reduce differences between groups. If subjects with a wider I.Q. range had been studied some of the effects might have been more evident.

Chapter VI

SUMMARY

The purpose of this research was to study changes in the problem solving processes of high school students as a function of a carefully defined program of training. Training was defined as "the development of a student's won methods of handling problems by helping him to become aware of his methods".

It was hypothesized that "problem solving processes may be evaluated better by studying the sequence of questions asked by a subject as he solves a problem than by evaluating his solution".

The first part of this research consisted in the development of problems constructed in such a way that the successive steps followed to reach a solution could be observed. A total of 26 problems were prepared.

The feature common to all of them is that subjects had to seek out information by asking those questions that they consider necessary to reach the solution. In some of the problems a set of questions was presented to the subjects and in others they generated their own questions.

In all cases the experimenter kept a record of the number and order of the questions asked.

Many of the problems had to be developed as the research proceeded in order to explore what appeared to be significant clues. This is due to the fact that the instruments used in this research as well as some of the basic assumptions are different from those commonly used.

In this sense the study has also been exploratory.

The general outline of the research was as follows: Selection of 50 high school students paired in terms of I.Q. (above 120 in the Henman-Nelson Test). Twenty-five students were randomly assigned to the control group and twenty-five to the experimental group.

Each subject of the experimental group went through a "training" period that consisted in attempting from 18 to 23 problems and in discussing his performance with the experimenter. This part of the experiment lasted throughout the second semester of 1961. Each testing session for each subject took place at approximately every 2 weeks and lasted about forty minutes.

Both the experimental and the control groups were administered 2 problems at the beginning of the experiment and 2 problems at the end. These problems were identical for both groups.

The data was analyzed in terms of changes in test performance as the experiment developed. The following scoring methods were used a) number of correct solutions, b) number of questions asked, c) characterization of process in terms of performance (see text) and d) verbal reports. Other scoring procedures were developed as part of this research.

A comparison was made between the performance of the experimental and control groups in the 2 tests administered at the beginning and at the end of the experiment.

It was found that in terms of problem solving performance the difference between the experimental and control groups was greater at the end of the experiment than at the beginning. This refers mostly to the apparently greater agreement between the experimental subjects in what they consider to be useful questions to be asked to solve the problems.

It was also found that the experimental subjects showed a greater gain in mathematics grades than the control subjects. This was not apparent for grades in English, Latin and average academic subjects.

The problems were designed to exemplify several aspects of problem solving and to observe the related behavior and behavior changes on the part of the subjects.

1) Subjects are guided by the more obvious features of the problem in devising a plan for its solution. The use of less obvious leads is quite difficult for some.

2) Subjects displayed varying degrees of hypothesis testing and formulation.

3) The judicious use of negative information, i.e., the willingness to modify one's thinking and to determine what convictions need to be given up, is a serious problem with many subjects, that does, however, appear improvable.

4) The effect of previously correct approaches on subsequent problems seems to diminish as the variety of solutions of previous, similar problems is increased.

5) The use of principles, once discovered, for further classificatio appears to involve a different process than their use in the creation of

new exemplars.

6) The terms in which the logical structure of a problem is clothed, whether familiar or unfamiliar, seems to be a major determiner of its difficulty as well as to orient the subjects towards particular approaches.

References

1. Blatt, S. J. Patterns of Cardiac Arousal During Complex Mental Activity, *Journal of Abnormal and Social Psychology.*, 1961 (In Press).

2. Bloom, B. S. and Broder, L. G., *Problem Solving Processes of College Students*, Chicago: The University of Chicago Press, 1950.

3. Bruner, J., *The Process of Education*, Cambridge: Harvard University Press, 1960.

4. Bruner, J. S., Goodnow, J. J., and Austin, G. A., *A Study of Thinking*, John Wiley & Sons, New York, 1960.

5. Bryan, G. L, *The Automasts: An Automatically Recording Test of Electronic Trouble Shooting*, Los Angeles: University of Southern California, Report No. 11, 1954

6. Buswell, G. T. and Kersh, B. Y., *Patterns of Thinking in Solving Problems*, University of California Publications in Education, University of California Press, Berkeley and Los Angeles, 12, 2, 1956.

7. Devane, J., Rimoldi, H. J. A., and Haley, J., Characteristics of the Approach of Physicians to the Test of Diagnostic Skills, Chicago: Loyola University, Loyola Psychometric Laboratory, 1959. (pub. No. 6

8. Devane, J., Rimoldi, H. J. A., and Haley, J., A Comparison of the Performance of Two Student Groups and Physicians in the Test of Diagnostic Skills, Chicago: Loyola University, Loyola Psychometric Laboratory, 1960. (pub. No. 10).

9. Duncan, C. P., Recent Research on Human Problem Solving, *Psychological Bulletin*, 56, 1959.

10. Feierabend, R., and DuBois, P., *Psychological Problems and Research Methods in Mathematics Training*, St. Louis, Washington University, 1959.

11. Glaser, R., Damrin, D. E., and Gardner, F. M., *The Tab Item Technique for the Measurement of Proficiency in Diagnostic Problem Solving Tasks*, Champaign: University of Illinois, College of Education, Bureau of Research and Service, 1952.

12. Gunn, H., Appraisal of Personality Parameters in Terms of Processes, Ph.D. Dissertation, Chicago: Loyola University, 1961.

13. Haley, J. V., The Effect of Learning on Performance in the Test of Diagnostic Skills, Chicago: Loyola University, Psychometric Laboratory, 1960. (pub. No. 11).

14. John, E. R., *Contributions to the Study of the Problem Solving Process.*, *Psychological Monographs*, 71, 18, 1957.

15. John, E. R., and Rimoldi, H. J. A., Sequential Observation of Complex Reasoning., *The American Psychologist*, 470, 1955 (Abstract).

16. Johnson, D. M., Formulation and Reformulation of Figure Concepts, *The American Journal of Psychology*, LXXIV, 1961.

17. Majewska, Sister M. Canisia, C.STF.N., A Study of Mathematical Ability As Related to Reasoning and Use of Symbols, Ph.D. Dissertation, Chicago: Loyola University, Loyola Psychometric Laboratory, 1960 (pub. No. 12)., *Educational and Psychological Measurement* (In press).

18. Mc Nemar, Q., *Psychological Statistics*, John Wiley & Sons, Inc., New York, 1955.

19. Mohrbacher, J., Interdisciplinary Evaluation of Organic Factors in the Psychopathology of Children, Ph.D. Dissertation, Chicago: Loyola University, 1961.

20. Rimoldi, H. J. A., A Technique for the Study of Problem Solving, *Educational and Psychological Measurement*, 15. 4.,1955.

21. Rimoldi, H. J. A., Problem Solving as a Process, *Educational and Psychological Measurement*, 20. 3., 1960.

22. Rimoldi, H. J. A., The Test of Diagnostic Skills, *Journal of Medical Education*, 36. 1., 1961.

23. Rimoldi, H. J. A., L'étude des Processus Psychologiques, *Le Travail Humain* , 1961 (In Press). Chicago: Loyola University, Loyola Psychometric Laboratory (pub. No. 19)., The Study of Psychological Processes, 1961.

24. Rimoldi, H. J. A., Devane, J. R., Inner Organization of the Clinical Diagnostic Process as Appraised by the Test of Diagnostic Skills, Chicago: Loyola University, Loyola Psychometric Laboratory, 1958. (pub. No. 4).

25. Rimoldi, H. J. A., Haley, J. V., Sequential Evaluation of Problem Solving Processes, Chicago: Loyola University, Loyola Psychometric Laboratory, 1961. (pub. No. 20) (In Press).

26. Rimoldi, H. J. A., Devane, J. R., and Grib, T. F., Testing Skills in Medical Diagnosis, Chicago: Loyola University, Loyola Psychometric Laboratory, 1958. (pub. No. 2).

27. Rimoldi, H. J. A., Devane, J. R., and Haley, J. V., Characterization of Processes, *Educational and Psychological Measurement*, 22, 2, 1961. Chicago: Loyola University, Loyola Psychometric Laboratory, 1959. (pub. No. 8).

28. Rimoldi, H. J. A., Majewska, Sister M. Canisia, C.S.F.N., Decision Processes in Mathematical Thinking, Proposal of Research sponsored by the National Science Foundation., 1961.

29. Studies in Mathematics Education: A Brief Survey of Improvement Programs for School Mathematics, Chicago: Scott, Foresman and Company, 1959.

30. Studies in Mathematics Education: A Brief Survey of Improvement Programs for School Mathematics, Chicago: Scott, Foresman and Company, 1960.

31. Tate, M. W., Stanier, B., Harootunian, B., Differences Between Good And Poor Problem Solvers., School of Education, University of Pennsylvania, Philadelphia, Pennsylvania, 1959.

32. Tabor, A. B., Process Analysis of Rorschach Interpretation, Ph.D. Dissertation., Chicago: Loyola University, 1959.

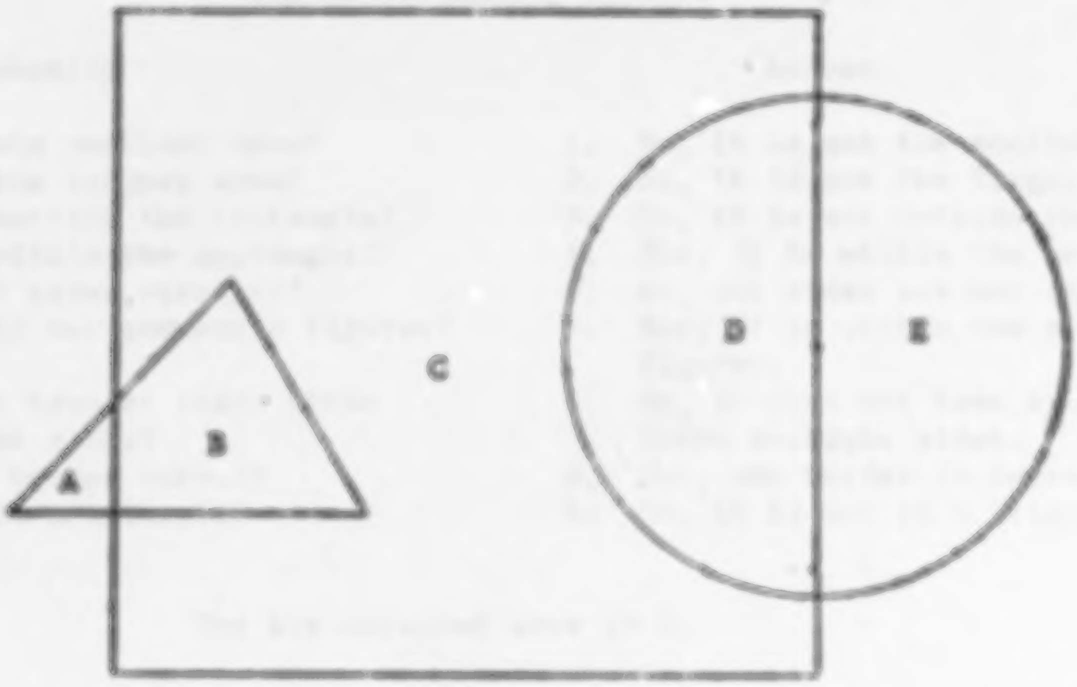
33. Werdelin, I., The Mathematical Ability, Lund, 1958.

Instructions for Problem 1

Figure I is composed of an overlapping rectangle, triangle, and circle, forming the areas A through E. You will notice that an area is any part of the figure that does not have a line through it. B, C, and D, for example, are separate areas, even though they are part of the same rectangle.

One of the areas has been picked at random. Your task is to find out which one it was. To find the particular area, you must discover enough facts about it so that it can be distinguished from the other areas. You may discover these facts by using any of the questions you like to arrive at the answer.

Proceed by reading over all the questions. Decide on the first question you would like to have answered and write its number on the page provided. Then withdraw the card from the folder, and read the answer on the back of the card. After having read the answer, decide on the next question you would like to have answered. Write down its number and withdraw the card. When you are satisfied that you have arrived at the answer, stop drawing cards, and write down your answer. Remember, you may draw as many cards as you need to find the correct area, but don't draw any more than you need.



Problem 1

Questions Employed in Problem 1
And Corresponding Answers

Question	Answer
1. Is it the smallest area?	1. No, it is not the smallest area.
2. Is it the largest area?	2. No, it is not the largest area.
3. Is it outside the rectangle?	3. No, it is not outside the rectangl
4. Is it within the rectangle?	4. Yes, it is within the rectangle.
5. Are all sides straight?	5. No, all sides are not straight.
6. Is it in two geometric figures?	6. Yes, it is within two geometric figures.
7. Does it have at least three straight sides?	7. No, it does not have at least three straight sides.
8. Is one border curved?	8. Yes, one border is curved.
9. Is it in a triangle?	9. No, it is not in a triangle.

Solution: The pre-selected area is D.

Appendix I

Instructions for Problem #2

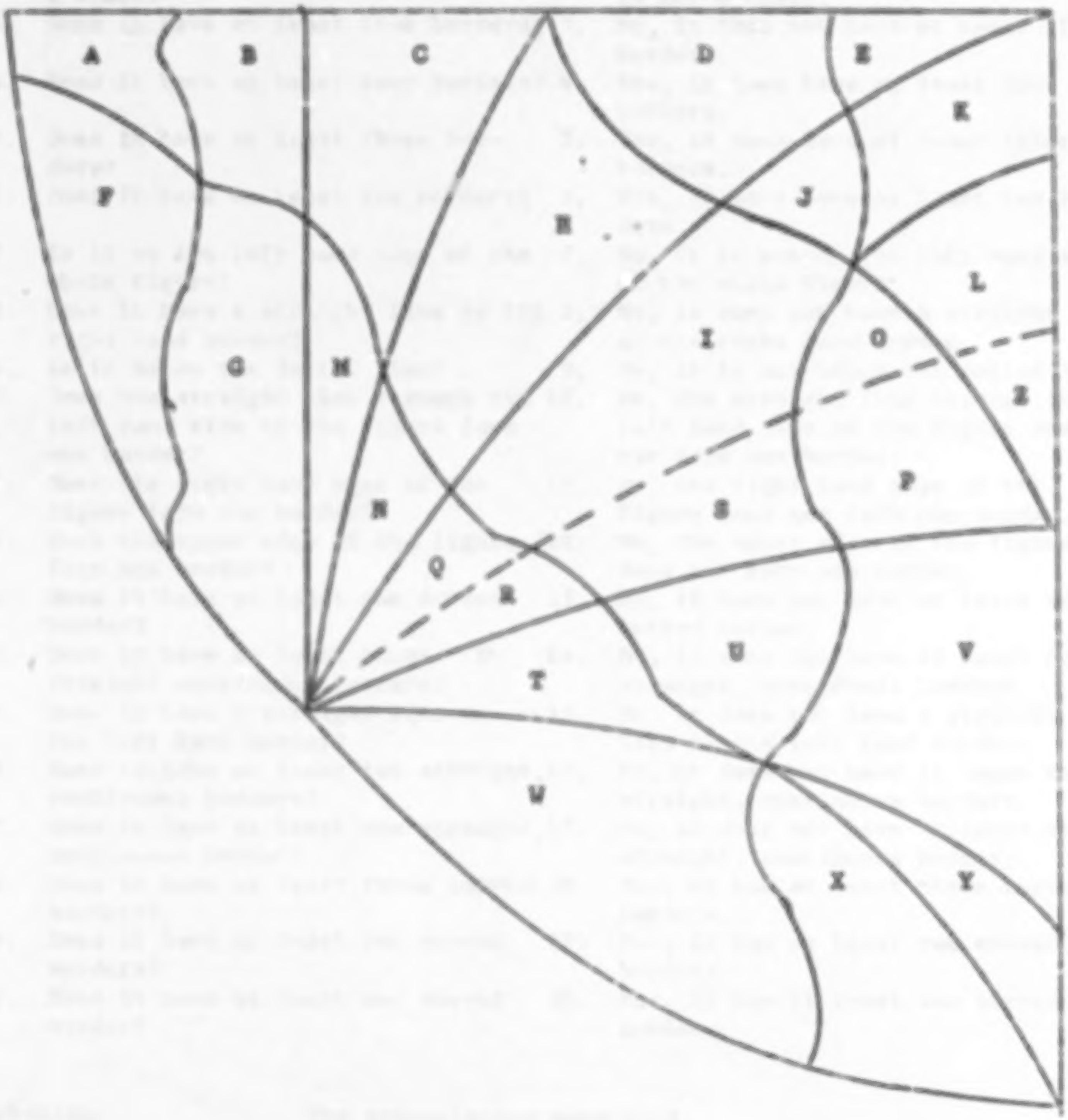
The figure is composed of straight, curved, and dotted lines. Lines that are not dotted are called continuous lines. You may notice that a line that is mostly curved may appear straight for a short distance; you must therefore look at the whole line, and not just a small part of it to decide whether it is straight or curved.

The areas are labeled A through Z. One of them has been picked at random. Your task is to determine which one it was.

To find the particular area, you must discover enough facts about it so that it can be distinguished from the other areas. You may discover these facts by using any of the questions you like to arrive at the answer.

Proceed by reading over all the questions. Decide on the first question you would like to have answered and write its number on the page provided. Then withdraw the card from the folder, and read the answer on the back of the card. After having read the answer on the back of the card, decide on the next question you would like to have answered. Write down its Number and withdraw the card. When you are satisfied that you have arrived at the answer, stop drawing cards, and write down your answer. Remember, you may draw as many cards as you need to find the correct area, but don't draw any more than you need.

Problem 2


A
B
C
D
E
K
F
J
H
L
I
O
G
M
Z
N
P
S
Q
R
U
V
T
W
X
Y

Appendix I

Questions Employed in Problem 2
And Corresponding Answers

Question	Answer
1. Is it one of the larger areas?	1. Yes, it is one of the larger areas.
2. Is the letter designating the area a vowel?	2. No, the letter designating the area is not a vowel.
3. Does it have at least five borders?	3. No, it does not have at least five borders.
4. Does it have at least four borders?	4. Yes, it does have at least four borders.
5. Does it have at least three borders?	5. Yes, it does have at least three borders.
6. Does it have at least two borders?	6. Yes, it does have at least two borders.
7. Is it on the left hand edge of the whole figure?	7. No, it is not on the left hand edge of the whole figure.
8. Does it have a straight line as its right hand border?	8. No, it does not have a straight line as its right hand border.
9. Is it below the dotted line?	9. No, it is not below the dotted line.
10. Does the straight line through the left hand side of the figure form one border?	10. No, the straight line through the left hand side of the figure does not form one border.
11. Does the right hand edge of the figure form one border?	11. No, the right hand edge of the figure does not form one border.
12. Does the upper edge of the figure form one border?	12. No, the upper edge of the figure does not form one border.
13. Does it have at least one dotted border?	13. No, it does not have at least one dotted border.
14. Does it have at least three straight continuous borders?	14. No, it does not have at least three straight, continuous borders.
15. Does it have a straight line as its left hand border?	15. No, it does not have a straight line as its left hand border.
16. Does it have at least two straight, continuous borders?	16. No, it does not have at least two straight, continuous borders.
17. Does it have at least one straight, continuous border?	17. No, it does not have at least one straight, continuous border.
18. Does it have at least three curved borders?	18. Yes, it has at least three curved borders.
19. Does it have at least two curved borders?	19. Yes, it has at least two curved borders.
20. Does it have at least one curved border?	20. Yes, it has at least one curved border.

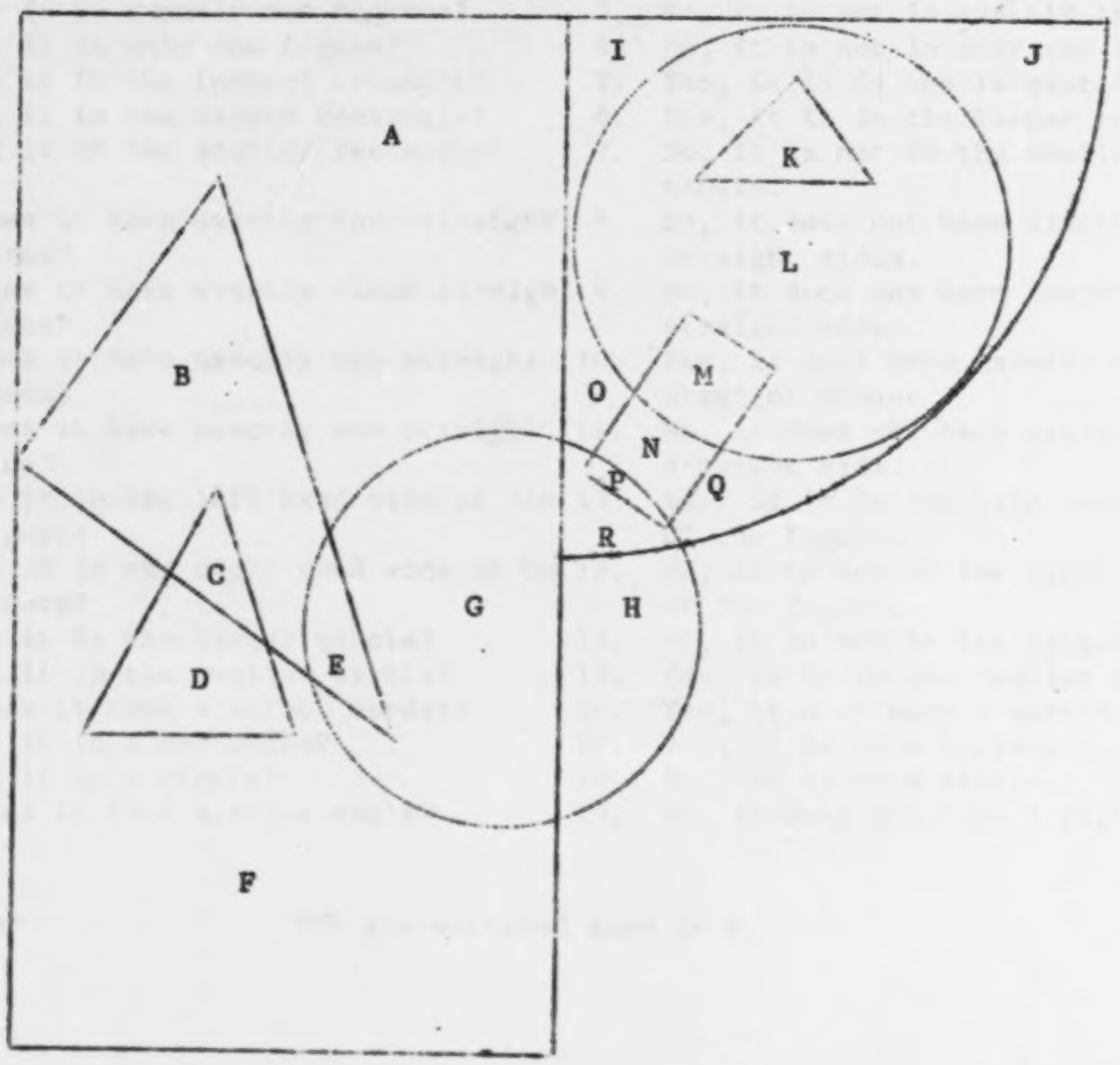
Solution: The pre-selected area is H.

Instructions for Problem 3

The figure here is made of overlapping geometric figures. You will notice that an area is any enclosed part of the figure that does not have a line through it. One of the areas has been picked at random. Your task is to find out which one it was. To find the particular area, you must discover enough facts about it so that it can be distinguished from other areas. You may discover these facts by using any of the questions you like.

Proceed by reading over all the questions. Then decide on the first question you would like to have answered, and write its number on the page provided. Then take the card from the folder, and read the answer on the back of the card. After having read the answer, decide on the next question you would like to have answered. Write down its number, and take the card from the folder. When you are satisfied that you have arrived at the answer, stop drawing cards, and write down your answer. Remember, you may use as many of the cards as you need to find the correct area, but don't draw any more than you need.

Problem 3


A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R

Appendix I

Questions Employed in Problem 3
And Corresponding Answers

Question	Answer
1. Is it in exactly four figures?	1. No, it is not in exactly four figures.
2. Is it in exactly three figures?	2. Yes, it is in exactly three figures.
3. Is it in exactly two figures?	3. No, it is not in exactly two figures
4. Is it in only one figure?	4. No, it is not in only one figure.
5. Is it in the largest triangle?	5. Yes, it is in the largest triangle.
6. Is it in the larger rectangle?	6. Yes, it is in the larger rectangle.
7. Is it in the smaller rectangle?	7. No, it is not in the smaller rectangle.
8. Does it have exactly four straight sides?	8. No, it does not have exactly four straight sides.
9. Does it have exactly three straight sides?	9. No, it does not have exactly three straight sides.
10. Does it have exactly two straight sides?	10. Yes, it does have exactly two straight sides.
11. Does it have exactly one straight side?	11. No, it does not have exactly one straight side.
12. Is it in the left hand side of the figure?	12. Yes, it is in the left hand side of the figure.
13. Is it in the right hand side of the figure?	13. No, it is not in the right hand side of the figure.
14. Is it in the larger circle?	14. No, it is not in the larger circle.
15. Is it in the smaller circle?	15. Yes, it is in the smaller circle.
16. Does it have a curved border?	16. Yes, it does have a curved border.
17. Is it in a rectangle?	17. Yes, it is in a rectangle.
18. Is it in a circle?	18. Yes, it is in a circle.
19. Does it have a right angle?	19. No, it does not have a right angle.

Solution: The pre-selected area is E.

Instructions for Problem 4

There are three columns and ten rows in the table. There is a word that belongs in each of the thirty boxes. Your task is to find the words that belong in each of the boxes with question marks.

The words in the first column follow a rule; that is, there will be some relationship between, for example, the words in the fifth and sixth boxes of the first column. The words in each of the rows are also related; that is, there is some connection between, for example, the words in the fifth box of the first column and the fifth box of the second column, and there will be another connection between the fifth box of the first column and the fifth box of the third column. The same will hold for each row.

To find the word that belongs in any one of the boxes with question marks, you must find out the rules to use. To find out these rules, you will need to know more words than the few that are given. Decide what box you would like to know about and then select the question that will give you that information. Write the word in the box. Then decide on the next box you would like to know about; select the question that will give you the information and write that word in the box. When you have decided on the words that belong in the boxes with question marks, and you are satisfied that they are correct, stop drawing cards and write in your answers.

Appendix I

Problem 4

1.	lamp	vamp	gone
2.	light	?	await
3.			
4.			?
5.			
6.	shoot	root	await
7.			
8.			?
9.	?		
10.			

Appendix I

Key to Problem 4

	A	B	C
1	lamp	vamp	gone
2	light	sight	await
3	torch	porch	await
4	flame	claim	await
5	fire	mire	gone
6	shoot	root	await
7	branch	ranch	planet
8	limb	grim	gone
9	leg	beg	tip
10	support	report	directs

Appendix I

Instructions for Problem 5

You wish to drive from Centerville to Peterston. The latter is due East of Centerville. There are three different roads, Highway Numbers 31, 27, and 9 which run from the first to the second city. You are going to start out on Highway 31.

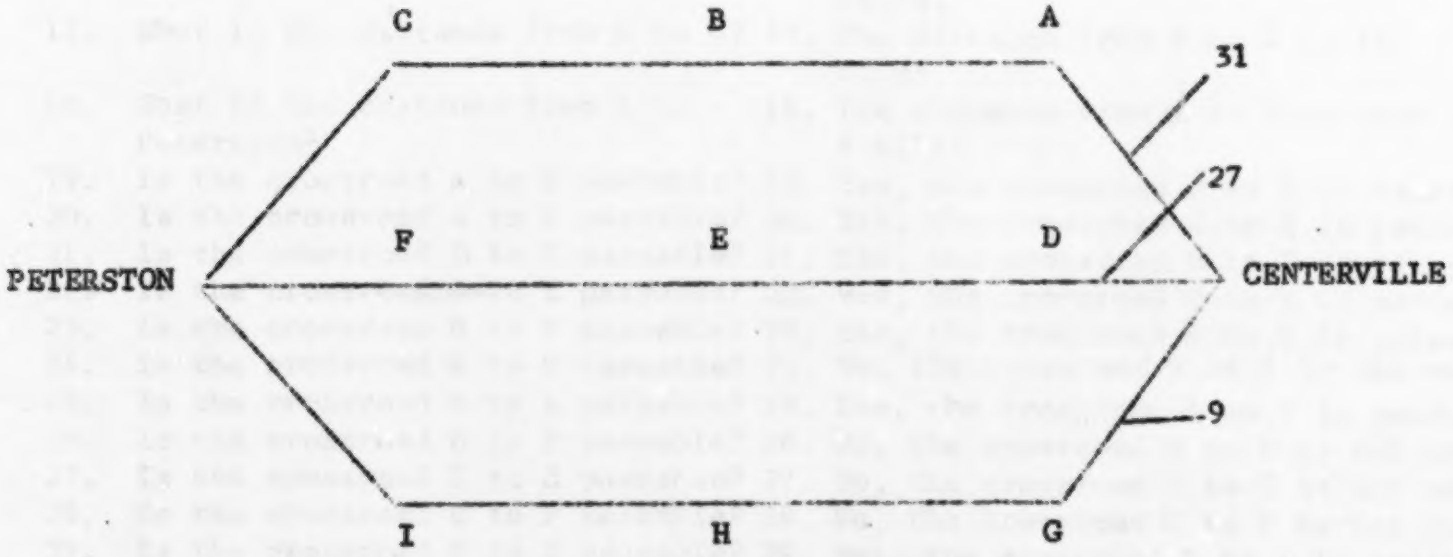
A recent storm has made each of the highways impassable in two places. However, since the highways are connected by crossroads, it may yet be possible to make the trip.

Your task will be to plan a route to Peterston, if this is possible, turning off at various towns, from one highway to another, in order to avoid the breaks in the highways.

Below is a rough sketch of the three roads from Centerville to Peterston. It is not a road map; for example, you could not find out the distances between any two towns from the sketch. Furthermore, none of the crossroads have been drawn in.

As you can see in the sketch, Highway 31 runs through the towns, A, B, and C, in this order, before reaching Peterston. Highway 27 runs through the towns D, E, and F, while Highway 9 runs through G, H, and I. Of course, all three highways meet in the town of Peterston.

To settle the matter, decide which question you want answered, and turn over the card. The answer to the question is on the reverse side. After reading the answer, decide on the next question, proceeding in this manner until you are satisfied with your solution. Select as many questions as you consider necessary and sufficient.



Appendix I

Questions Employed in Problem 5
And Corresponding Answers

Question	Answer
1. Where is the first break on Highway 31?	1. The first break on Highway 31 is 6 miles East of A.
2. Where is the second break on Highway 31?	2. The second break on Highway 31 is 4 miles West of C.
3. Where is the first break on Highway 27?	3. The first break on Highway 27 is 1 mile East of D.
4. Where is the second break on Highway 27?	4. The second break on Highway 27 is 10 miles East of D.
5. Where is the first break on Highway 9?	5. The first break on Highway 9 is 1 mil East of G.
6. Where is the second break on Highway 9?	6. The second break on Highway 9 is 2 miles East of I.
7. What is the distance from Centerville to A?	7. The distance from Centerville to A is 10 miles.
8. What is the distance from A to B?	8. The distance from A to B is 10 miles.
9. What is the distance from B to C?	9. The distance from B to C is 10 miles
10. What is the distance from C to Peterston?	10. The distance from C to Peterston is 8 miles.
11. What is the distance from Centerville to D?	11. The distance from Centerville to D is 6 miles.
12. What is the distance from D to E?	12. The distance from D to E is 8 miles.
13. What is the distance from E to F?	13. The distance from E to F is 8 miles.
14. What is the distance from F to Peterston?	14. The distance from F to Peterston is 4 miles.
15. What is the distance from Centerville to G?	15. The distance from Centerville to G is 10 miles.
16. What is the distance from G to H?	16. The distance from G to H is 10 miles.
17. What is the distance from H to I?	17. The distance from H to I is 10 miles.
18. What is the distance from I to Peterston?	18. The distance from I to Peterston is 8 miles.
19. Is the crossroad A to D passable?	19. Yes, the crossroad A to D is passable.
20. Is the crossroad A to E passable?	20. Yes, the crossroad A to E is passable.
21. Is the crossroad D to G passable?	21. Yes, the crossroad D to G is passable.
22. Is the crossroad G to E passable?	22. Yes, the crossroad G to E is passable.
23. Is the crossroad B to E passable?	23. Yes, the crossroad B to E is passable.
24. Is the crossroad B to F passable?	24. No, the crossroad B to F is not passal
25. Is the crossroad H to E passable?	25. Yes, the crossroad H to E is passable.
26. Is the crossroad H to F passable?	26. No, the crossroad H to F is not passal
27. Is the crossroad E to C passable?	27. No, the crossroad E to C is not passal
28. Is the crossroad C to F passable?	28. No, the crossroad C to F is not passab
29. Is the crossroad F to I passable?	29. Yes, the crossroad F to I is passable.

Solution: The open route is Centerville to A to E to H to I to F to Peterston.

Instructions for Problem 6

You have 100 objects, which for want of a better name, we will call A's. Some of the A's are also D's, just as a man who is a teacher may also be a golfer, or a book may be about history and have a red cover. You are to discover, if you can, how many of the A's are also D's, just as, if you knew ten Cub fans, you might discover how many of them are Democrats, or if you knew ten redheads, you might discover how many of them are left-handed.

Look over the questions, and decide which question you would like answered. Then turn over the card and read the answer on the back. Next, decide on the second question you would like answered, and turn over the card. Continue selecting questions until you are satisfied that you have the solution. Select as many of the questions as you feel you need. Don't ask more questions than you need, but don't hesitate to ask any one you consider useful. You may select the questions in any order you wish.

Questions Employed in Problem 6
And Corresponding Answers

Question	Answer
1. What is a B?	1. A B is a particular type of A.
2. What is a C?	2. A C is a particular type of A.
3. What is an E?	3. An E is something that is not an A.
4. How many A's are B's?	4. Fifty of the A's are B's.
5. How many A's are C's?	5. Fifty of the A's are C's.
6. How many A's are E's?	6. None of the A's are E's.
7. How many B's are also D's?	7. All the B's are D's.
8. How many C's are also D's?	8. One-half of the C's are D's.
9. How many E's are also D's?	9. None of the E's are also D's.

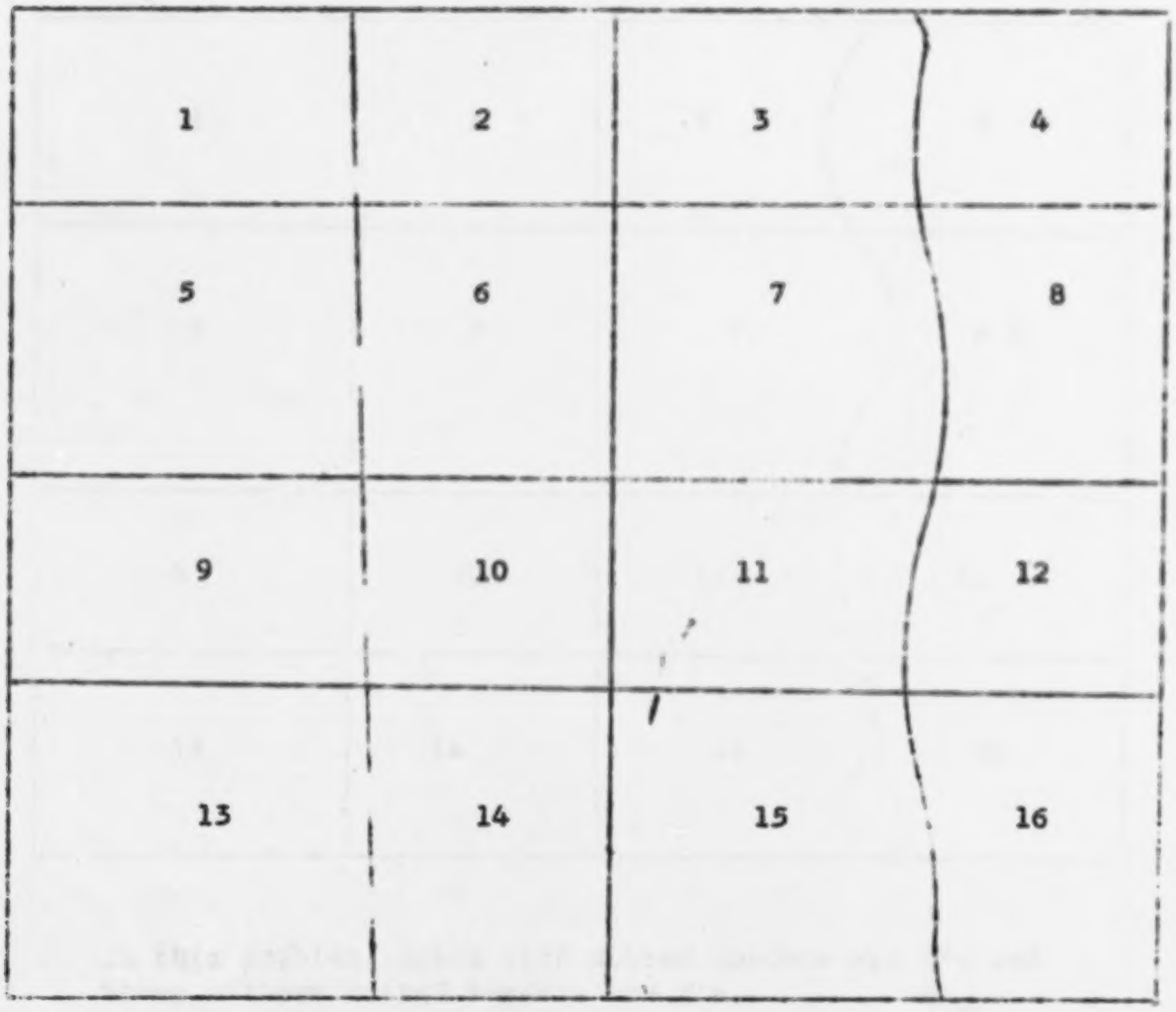
Solution: 75 A's are D's.

Instructions for Problems 7,8,9,11,12, 13, 14, 15, and 16

Each of the areas in the figure has a number. Each area could also have a letter, either X or Y. Your task is to decide whether Area Number 1 should have an X or a Y. You may ask about the letter that belongs in any of the other areas. When you have found out which areas should have X's and which areas should have Y's, you will know whether Number 1 should be an X or a Y.

Decide which area you would like to know about, and ask about it. Then decide which area you would next like to know about, and ask about it. When you have determined why some areas should be X's and some areas should be Y's, decide which Number 1 should be, and tell the examiner.

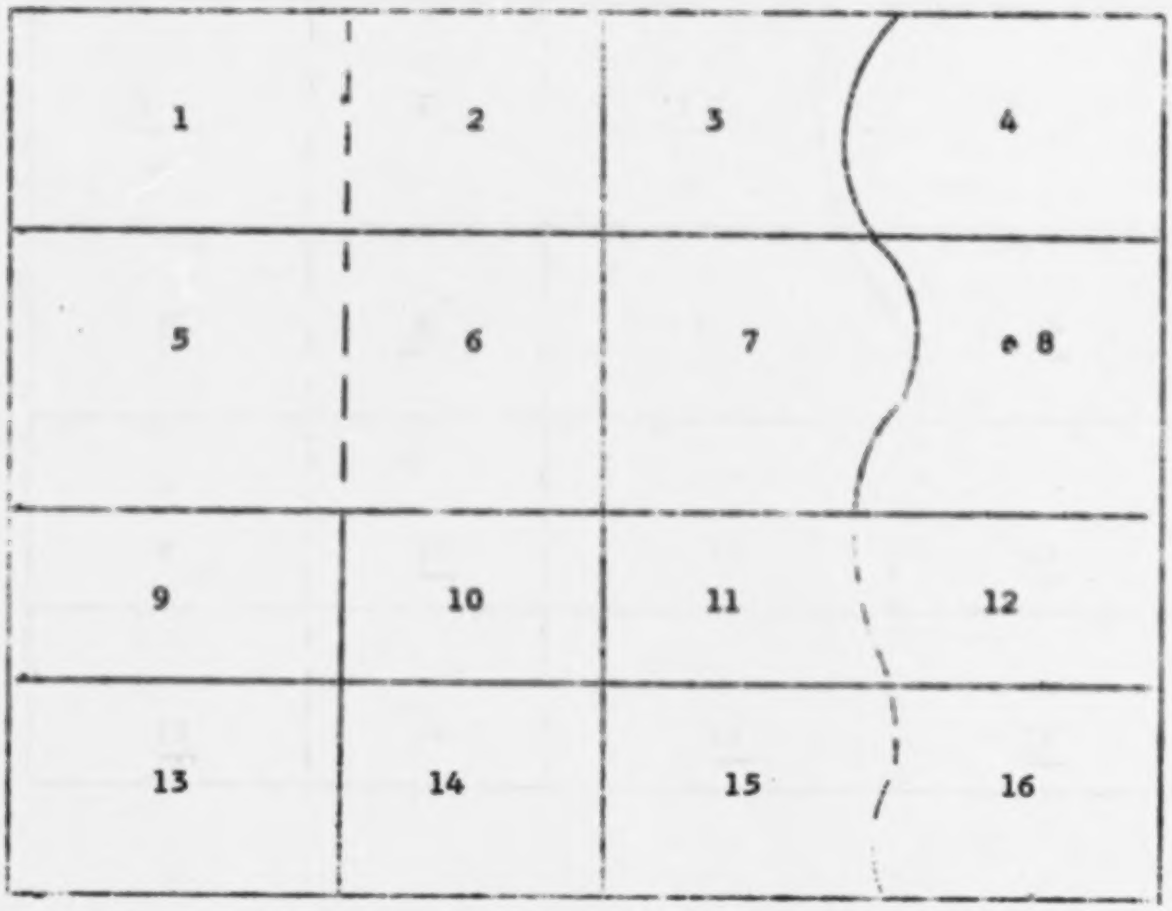
Problem 7



1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

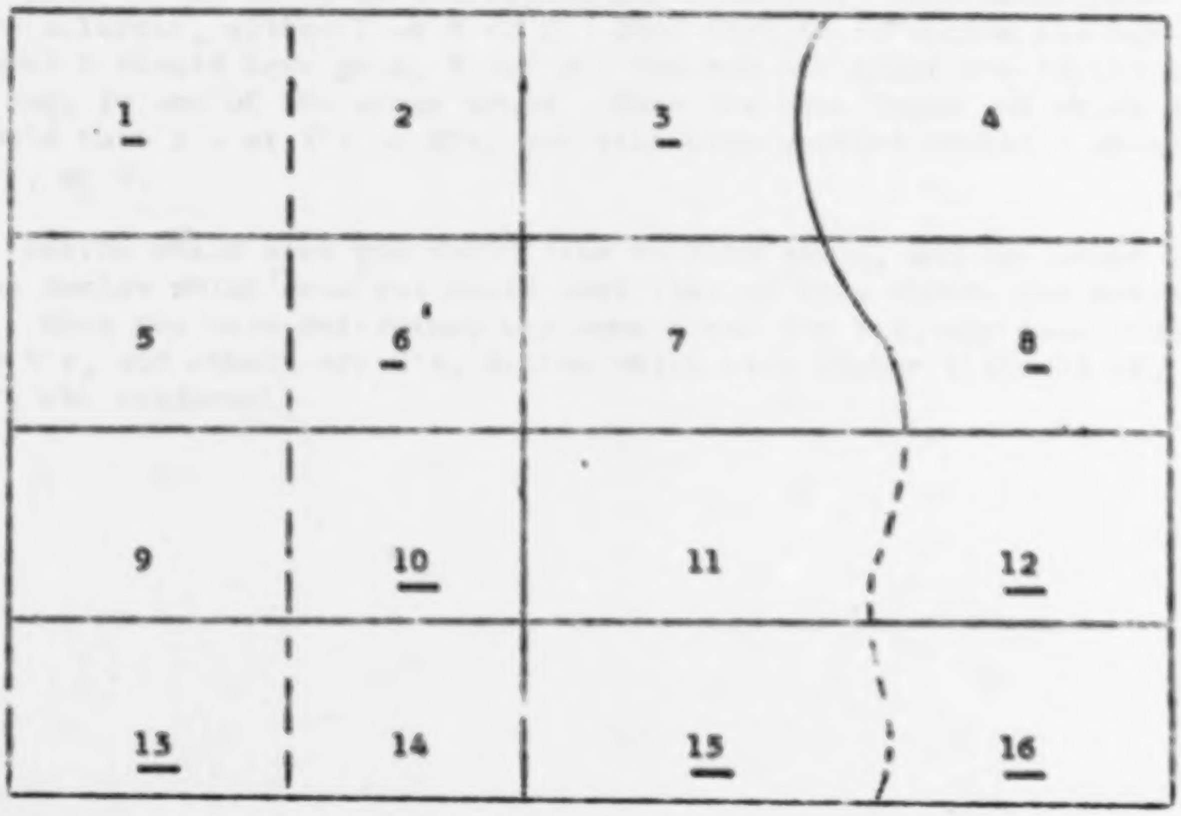
In this problem, odd numbers are X's, and even numbers are Y's.

Problem 8


1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

In this problem, areas with dotted borders are X's and those without dotted borders are Y's.

Problem 9


1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

Note: In the above figure numbers which are underscored were red in the original.

In this problem, areas with dotted borders and red numerals are X's.

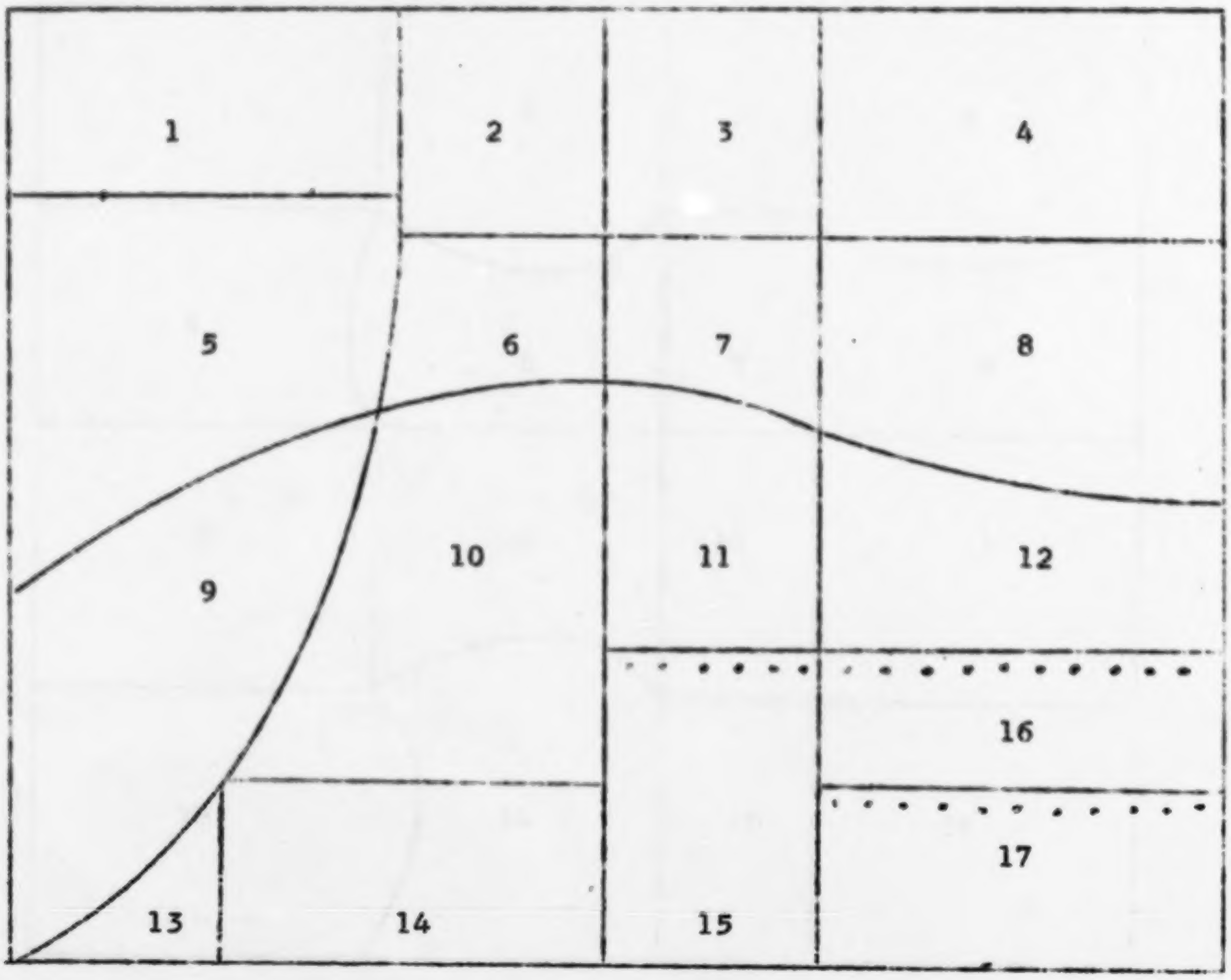
Instructions for Problem 10

Each of the areas in the figure has a number. Each area could also have a letter, either X or Y or Z. Your task is to decide whether area Number 1 should have an X, Y, or Z. You may ask about the letter that belongs in any of the other areas. When you have found out which areas should have X's or Y's or Z's, you will know whether Number 1 should be X, Y, or Z.

Decide which area you would like to know about, and ask about it. Then decide which area you would next like to know about, and ask about it. When you have determined why some areas are X's, why some others are Y's, and others are Z's, decide which area Number 1 should be, and tell the examiner.

Appendix I

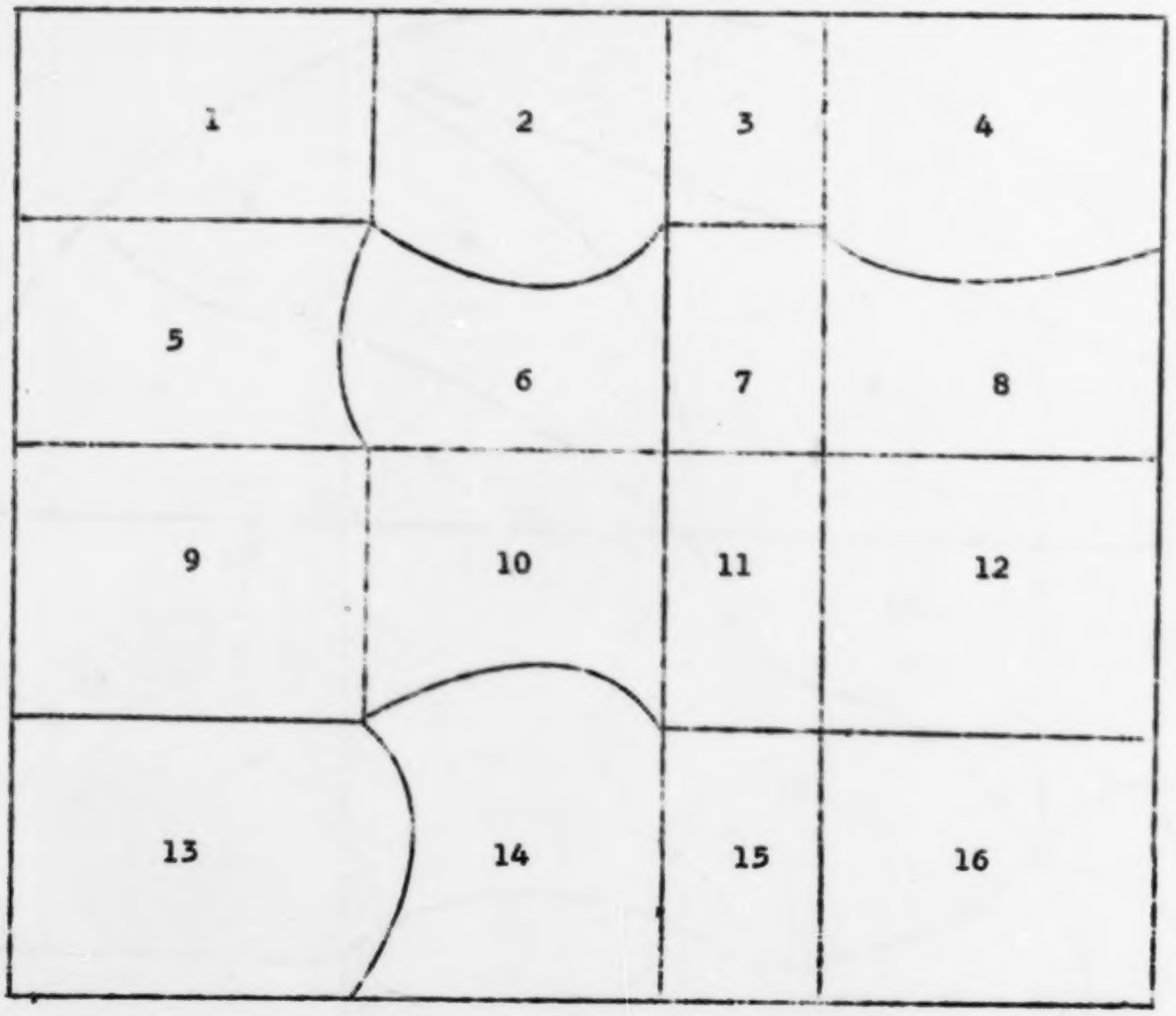
Problem 10



Note: In the above figure lines or portions of lines appearing with dots beside them indicates these to have been red in the original.

In this problem, Y's are areas where the base is greater than the height. X's are all other rectangles. Z's are any areas with curved borders.

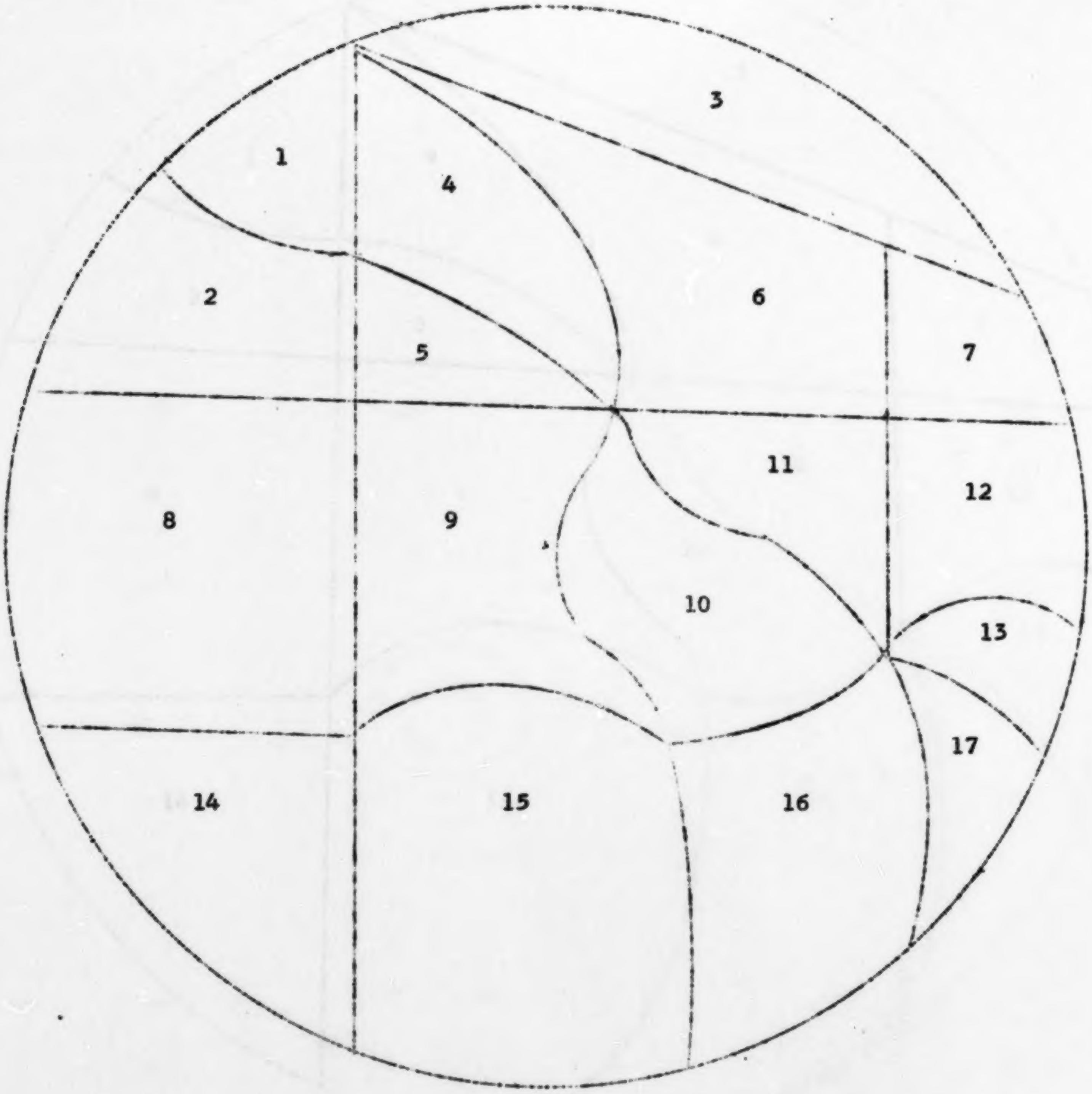
Problem 11


1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

In this problem, X's are all odd numbered areas with straight borders and Y's are all areas with curved borders and/or even numbers.

Appendix I

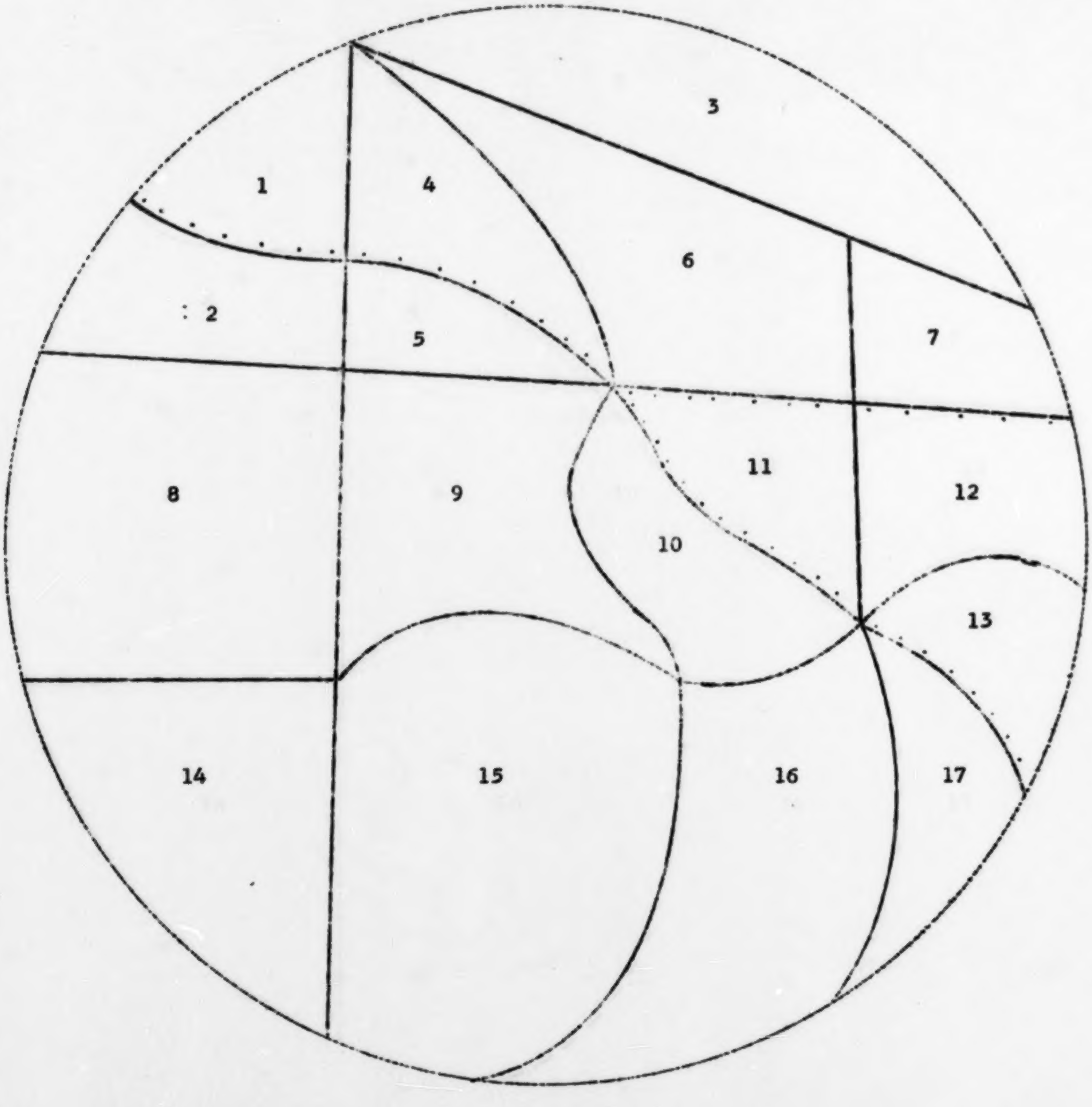
Problem 12


1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

In this problem all areas on the periphery are X's.

Appendix I

Problem 13

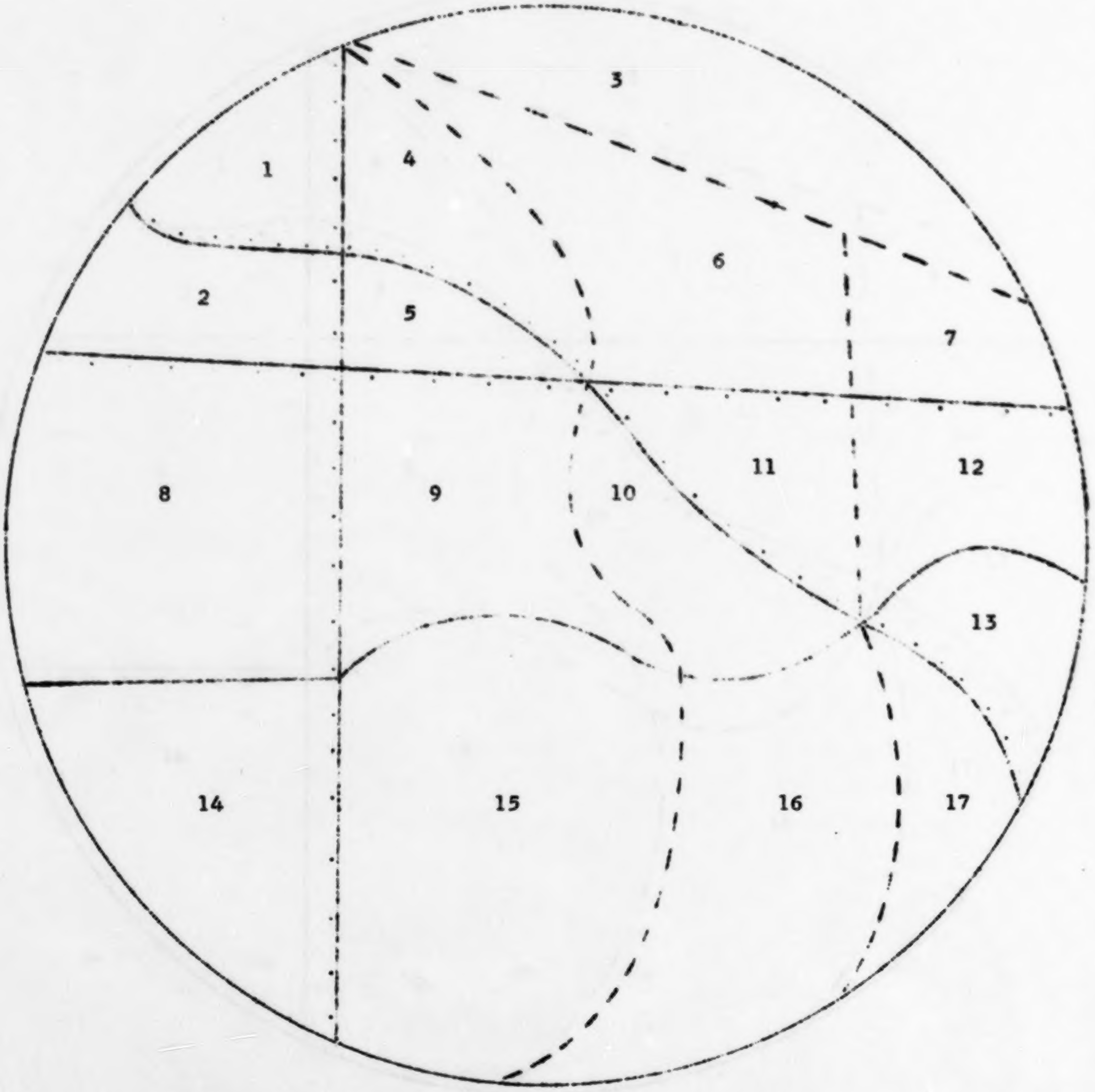

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

Note: In the above figure portions of lines appearing with dots beside them indicates these portions to have been red in the original.

In this problem all areas with red borders are X's.

Appendix I

Problem 14

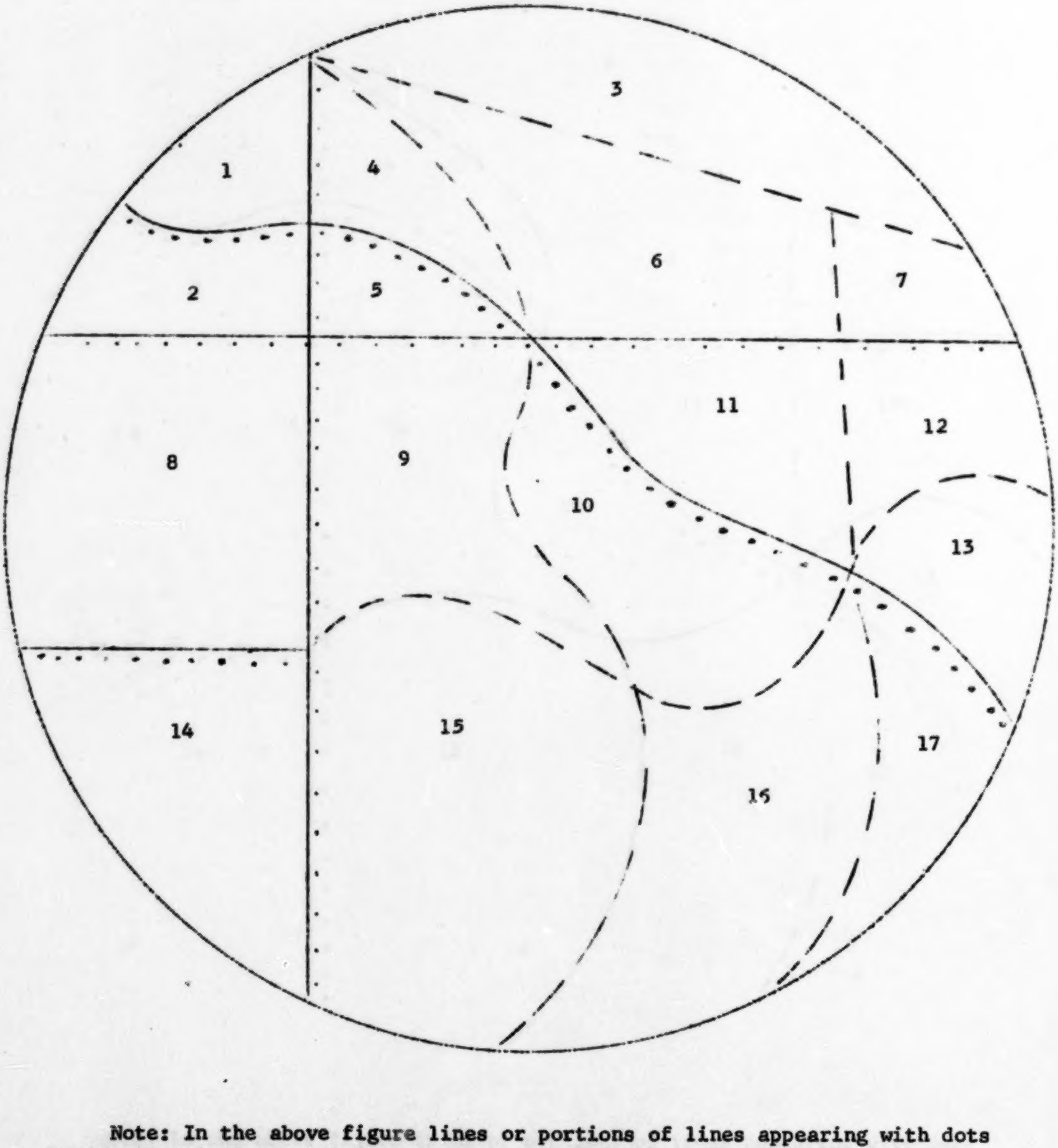

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

Note: In the above figure lines or portions of lines appearing with dots beside them indicates these to have been red in the original.

In this problem all areas with dotted borders are X's.

Appendix I

Problem 15

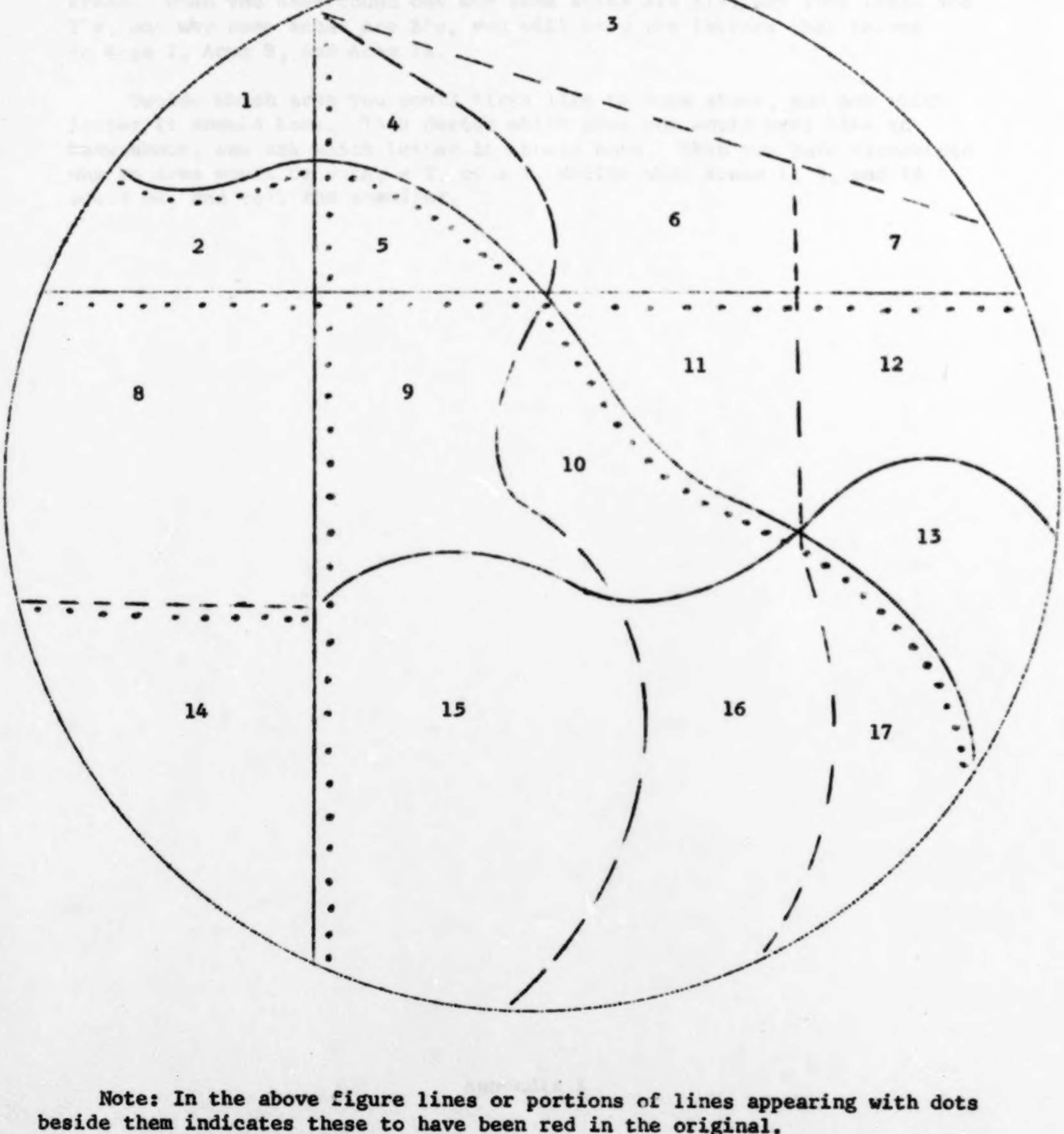

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

Note: In the above figure lines or portions of lines appearing with dots beside them indicates these to have been red in the original.

In this problem all areas with at least one red and one dotted side are X's.

Appendix I

Problem 16


1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

Note: In the above figure lines or portions of lines appearing with dots beside them indicates these to have been red in the original.

In this problem X's are areas with curved dotted sides and the Y's have no sides that are both dotted and curved.

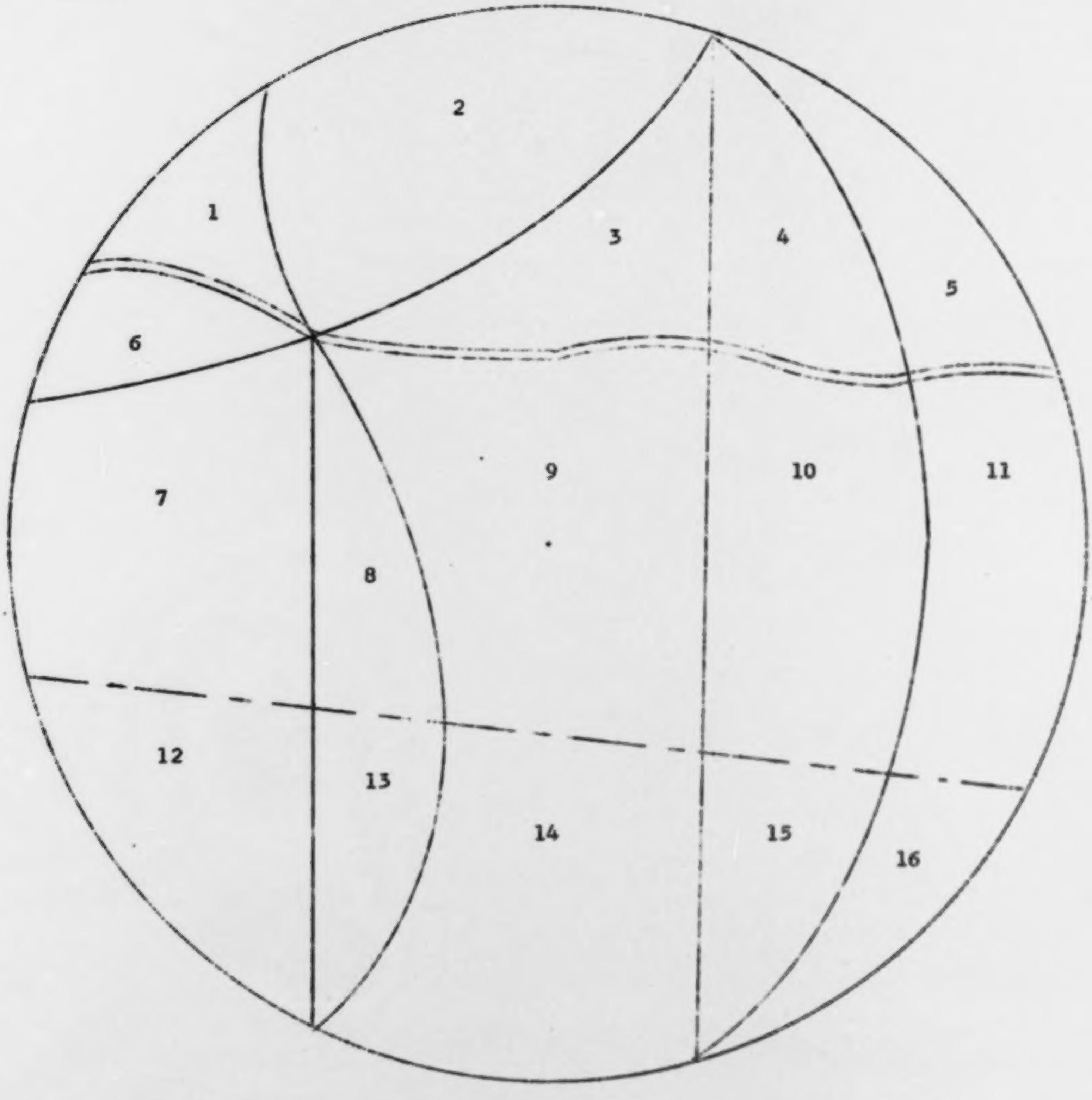
Appendix I

Instructions for Problem 17

Each of the areas in the figure has a number. Each area could also have a letter, either X, Y, or Z. Your task is to decide which letter belongs in Area 1, which letter belongs in Area 8, and which letter belongs in Area 14. You may ask about the letter that belongs in any of the other areas. When you have found out why some areas are X's, why some areas are Y's, and why some areas are Z's, you will know the letters that belong in Area 1, Area 8, and Area 14.

Decide which area you would first like to know about, and ask which letter it should have. Then decide which area you would next like to know about, and ask which letter it should have. When you have discovered why an area would be an X, a Y, or a Z, decide what Areas 1, 8, and 14 would be, and tell the examiner.

Problem 17


2
1
3
4
5
6
9
10
11
7
8
12
13
14
15
16

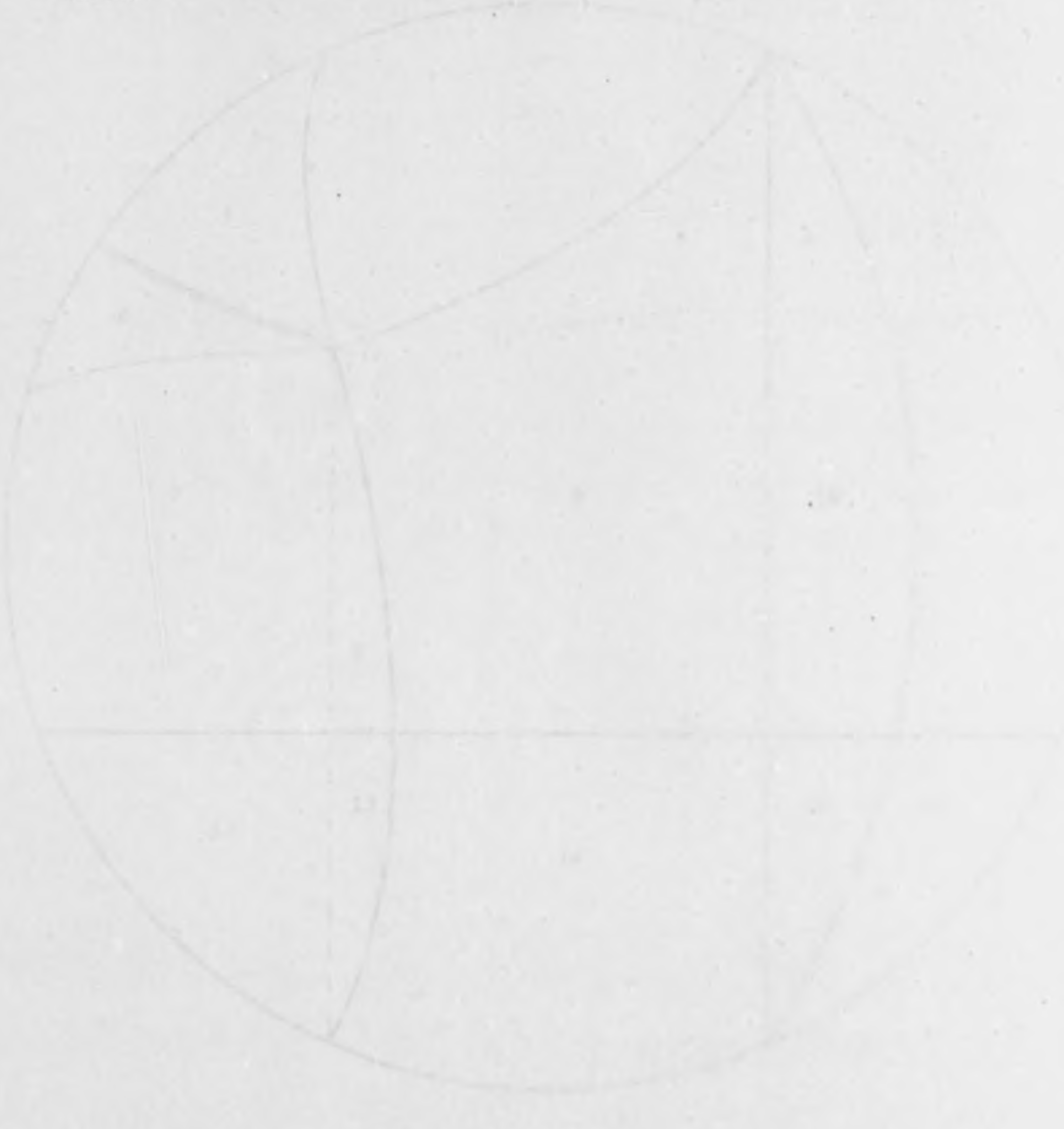
Note: In the above figure the double line represents a solid blue line in the original.

In this problem all areas above the blue line are X's, all areas between the blue and dotted lines are Y's, and those below the dotted lines are Z's.

Appendix I

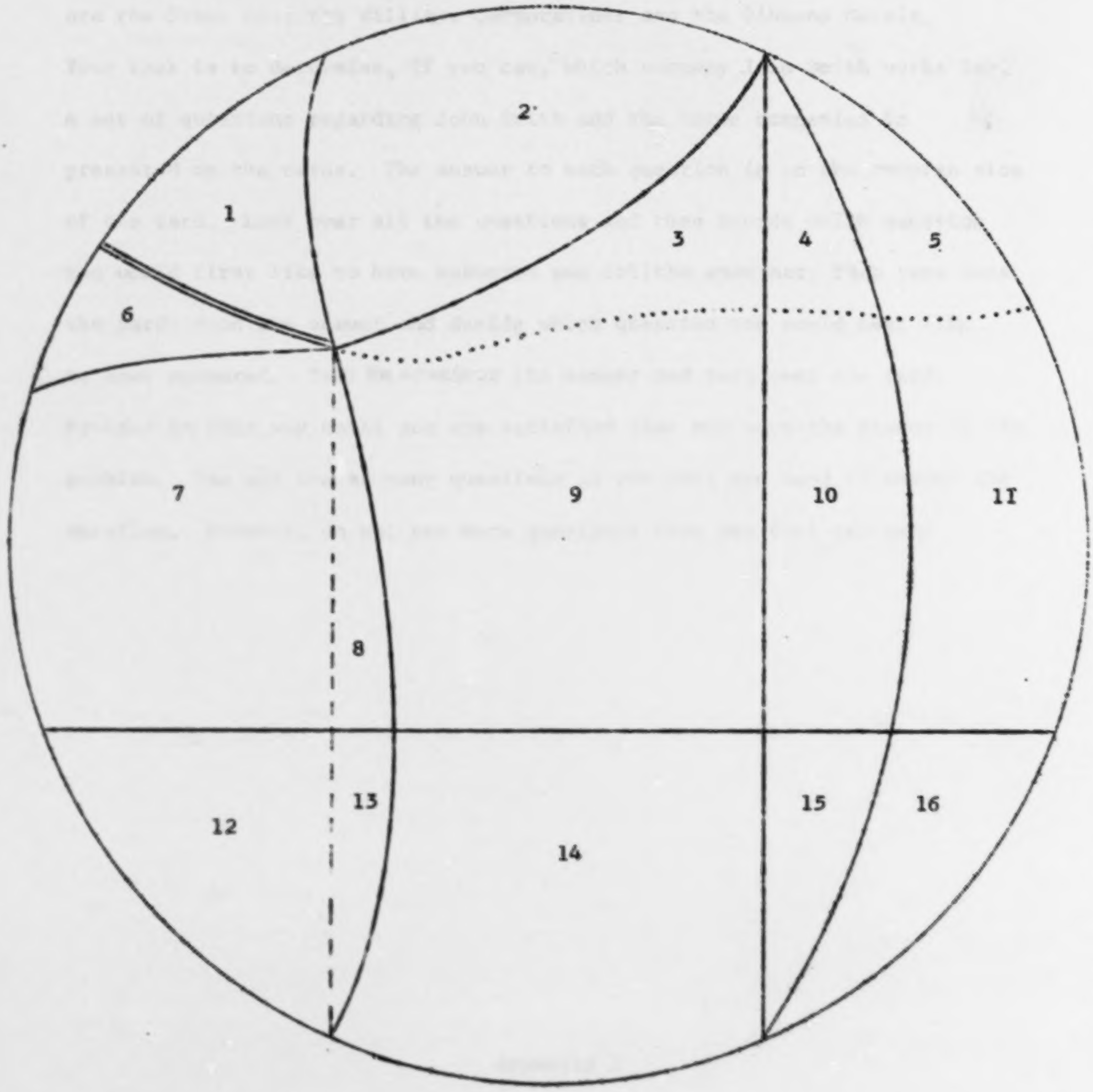
Instructions for Problem 18

Each of the areas in the figure has a number. Each area could also have a letter, either X, Y, or Z. Your task is to decide which letter belongs in Area 4, which letter belongs in Area 10, and which letter belongs in Area 14. You may ask about the letter that belongs in any of the other areas.



Appendix I

Problem 18



Note: In the above figure a double line represents a solid blue line in the original; a dotted line, a broken blue line.

In this problem all areas with a dotted blue border are X's, all areas with a dotted black border are Y's, all areas with solid border are Z's.

Appendix I

Instructions for Problem 19

John Smith lives in a small city that has only three companies. These are the Grant Co.; the Williams Corporation: and the Gibbons Metals.

Your task is to determine, if you can, which company John Smith works for.

A set of questions regarding John Smith and the three companies is presented on the cards. The answer to each question is on the reverse side of the card. Look over all the questions and then decide which question you would first like to have answered and tell the examiner. Then turn over the card; read the answer and decide which question you would next like to have answered. Tell the examiner its number and turn over the card. Proceed in this way until you are satisfied that you have the answer to the problem. You may use as many questions as you feel you need to answer the question. However, do not use more questions than you feel you need.

Questions Employed in Problem 19
And Corresponding Answers

	Question		Answer
1.	What is Mr. Smith's salary?	1.	His salary is $8000 per year.
2.	How much education has he had?	2.	He is a high school graduate. He wanted to go to college, but his family did not have enough money to send him.
3.	Is Mr. Smith married or single?	3.	Mr. Smith is married. He has three children.
4.	How old is Mr. Smith?	4.	He is 46 years old.
5.	What kind of job does he have?	5.	He is Supervisor of a loading dock.
6.	How long has he had his present job?	6.	He has had his job for 11 years.
7.	How long has he lived in his present home	7.	He has been in his present home for 6 years.
8.	What is the name of the city?	8.	The name of the city is Springton.
9.	Where in the city does Mr. Smith live?	9.	He lives at the North End of the cit near the high school.
10.	How many banks are there in the city?	10.	There are two banks; the First National and the State Bank.
11.	Does Mr. Smith have a bank account?	11.	No, Mr. Smith does not have a bank account.
12.	Where is the Grant Co.?	12.	The Grant Co. is just west of the city, off the main highway.
13.	Where is the Williams Corp?	13.	The Williams Corp is north of the city, just accross the river.
14.	Where is Gibbons Metals?	14.	Gibbons Metals is near the center of the city.
15.	How long has Grant and Co. been in the city?	15.	Grant Co. is the oldest firm in the city. It has been there over 60 yea
16.	How long has the Williams Corp. been in the city?	16.	The Williams Corp. has been in the city about 15 years.
17.	How long has Gibbons Metals been in the city?	17.	Gibbons Metals opened its plant in this city 4 years ago.
18.	How much education does Grant Co. require of its employees?	18.	The policy of Grant Co. in recent years has been to hire only college graduates. As a result, all employe under 40 are college graduates. Som of the employees over 40 are only high school graduates, while some others are college graduates.
19.	How much education does the Williams Corp. require of its employees?	19.	The Williams Corp. hires only college graduates for supervisory positions. Employees below supervisor may be high school graduates only.
20.	How much education does the Gibbons Metals require of its employees?	20.	Gibbons Metals has no policy on education. Its employees may have any amount of education.

Solution: Mr. Smith works for the Grant Co.

Appendix I

Instructions for Problem 20

There are ten students in a certain class in a boys' high school. One or more of them play on a varsity team. Your job is to decide how many of them play on the different team or teams and which teams or team it is.

To arrive at the answer, you will need to know different facts about the boys and the teams. Read over the questions on the cards and decide which one you would like answered. Tell the examiner its number and then take it out of the folder. After you have read the answer on the back of the card, decide which question you would next like to have answered. Proceed in this way until you are satisfied that you are able to answer the question.

Remember your task is to determine how many of the ten boys are on a varsity athletic team and which team (or teams).

Appendix I

Questions Employed in Problem 20
And Corresponding Answers

Question	Answer
1. Did any of the boys play on the Freshman teams?	1. Yes. Jim, Ted, Jack, and Bill playe on the Freshman Football team. Bill Pete, George and Ray played on the Freshman basketball team.
2. How tall is each of the boys?	2. Tom is 5'6"; Bill is 5'9"; Jack is 6'1"; Fred is 5'10"; George is 5'9"; Pete is 6'0"; Sam is 5'5"; Jim is 5'10"; Ted is 6'2"; and Ray is 6'0".
3. How much does each of the boys weigh?	3. Tom weighs 129; Bill 180; Jack 195; Fred 142; Pete 165; Sam 135; Jim 20 George 172; Ted 188; Ray 215.
4. What average does each boy have in school?	4. Tom has an average of 92; Bill's is Ted's is 86; Jack 89; Fred 80; Pete 8 Sam 95; Jim 90; George 88; and Ray
5. How old is each of the boys?	5. Tom is 16; Bill is 15; Jack is 15; Fred is 14; Pete is 15; Sam is 15; Jim is 16; George is 15; Ted is 14; and Ray is 15.
6. Do any of the boys have jobs in the afternoon?	6. Yes, Tom delivers for his Father's store, and Fred helps supervise in playground near his home.
7. Do any of the boys play in the band?	7. Yes, Sam and Pete both play the dru in the school band.
8. How many of them were on varsity teams last year?	8. None of them were on varsity teams last year.
9. May any Sophomore try out for the varsity?	9. Only students with the proper average, and who are at least 16 years old, may try out.
10. Is any one of the boys a class officer?	10. Yes, George is the class president.
11. What year are the boys in?	11. The boys are sophomores.
12. What are the names of the boys?	12. The boys' names are Tom, Bill, Jack Fred, Pete, Sam, Jim, George, Ted, and Ray.
13. Does a boy have a better chance of making the varsity team if he has played on the Freshman Team?	13. Yes, it helps a great deal. In fact every boy who played on the Freshman Team last year and is eligible for the varsity, is on the team.
14. Are there any Sophomores on the basketball team?	14. No, there are no Sophomores on the basketball team.
15. Are there any Sophomores on the football team?	15. Yes, they are linemen. The backfield is made up of Juniors and Seniors.
16. How tall are the basketball players?	16. Everyone on the basketball team is at least 6'0" tall.
17. What average must a boy have in his school subjects before he may play on a varsity team?	17. A varsity player must have an average of at least 87.

Questions Employed in Problem 20 And Corresponding Answers (Cont.)

Question	Answer
18. What afternoons do the teams practice?	18. The football team practices on Monday, Tuesday, and Thursday during the football season. The basketbal team practices every afternoon during the basketball season, except f the days on which a game is schedul
19. How big are the football players?	19. The linemen are all over 190 pounds. The backfield are all less than 190
20. How good are the varsity teams?	20. The football team won 4 and lost 3; the basketball team won 17 and lost
21. How many varsity teams are there?	21. There is a football team and a basketball team. There are no othe varsity teams.

Solution: Jim is on the football team.

Appendix I

Instructions for Problem 21

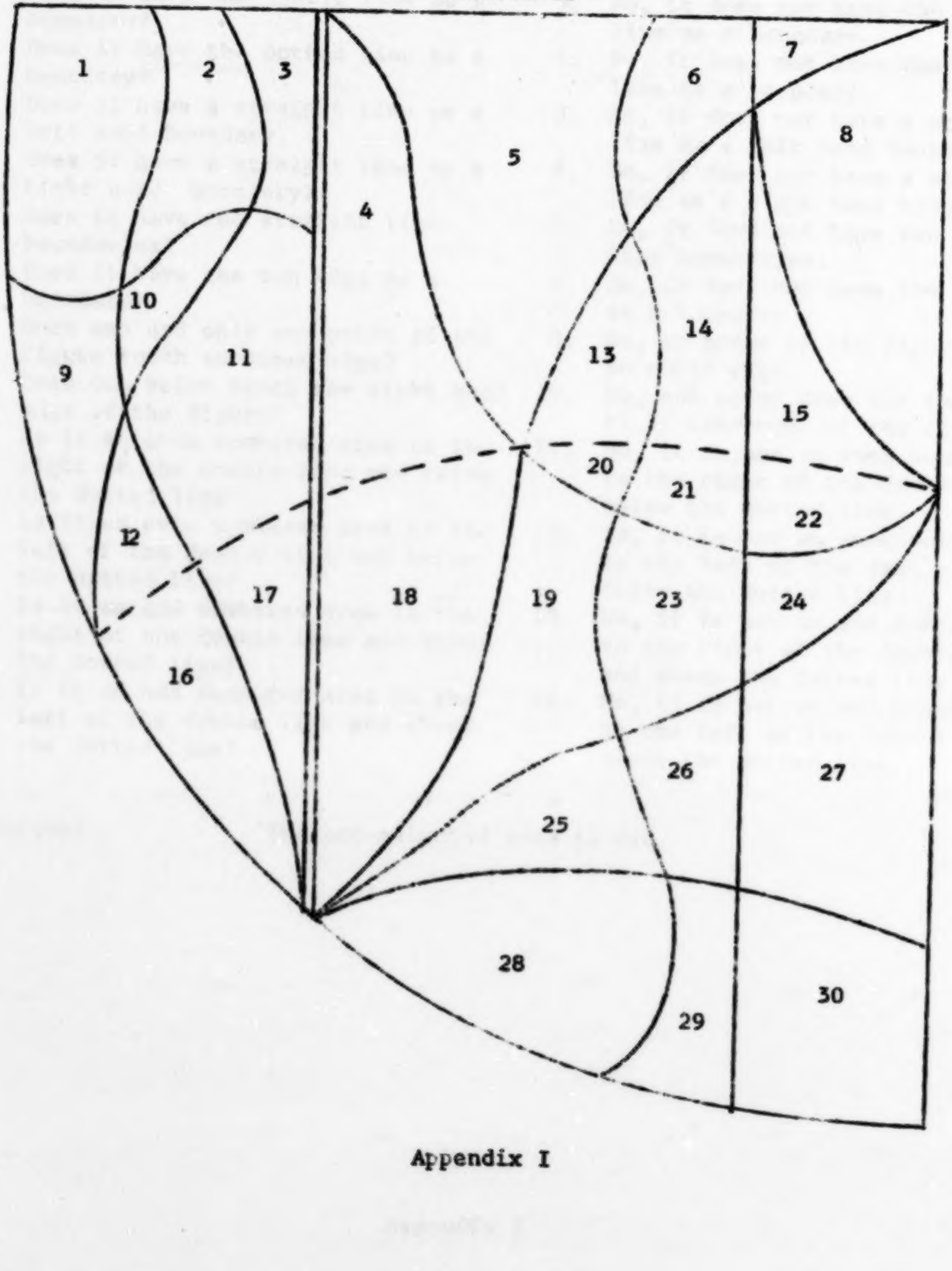
In this problem, there are 30 areas. Each area has been given a number and one of them has been picked at random. Your task is to determine which area it is.

To find the particular area, you must discover enough facts about it so that it can be distinguished from the other areas. You may discover these facts by using any of the questions you like to arrive at the answer.

Proceed by reading over all the questions. Decide on the first question you would like to have answered and write its number on the page provided. Then withdraw the card from the folder and read the answer on the back of the card. After having read the answer on the back of the card, decide on the next question you would like to have answered. Write down its number and withdraw the card. When you are satisfied you have arrived at the answer, stop drawing cards.

Appendix I

Problem 21


1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30

Appendix I

Questions Employed in Problem 21 And Corresponding Answers

	Question		Answer
1.	Is it the smallest area?	1.	No, it is not the smallest area.
2.	Is it the largest area?	2.	No, it is not the largest area.
3.	Does it have the double line as a boundary?	3.	No, it does not have the double line as a boundary.
4.	Does it have the dotted line as a boundary?	4.	No, it does not have the dotted line as a boundary.
5.	Does it have a straight line as a left hand boundary.	5.	No, it does not have a straight line as a left hand boundary.
6.	Does it have a straight line as a right hand boundary?	6.	No, it does not have a straight line as a right hand boundary.
7.	Does it have two straight line boundaries?	7.	No, it does not have two straight line boundaries.
8.	Does it have the top edge as a boundary?	8.	No, it does not have the top edge as a boundary.
9.	Does one and only one point of the figure touch an outer edge?	9.	No, no point of the figure touches an outer edge.
10.	Does one point touch the right hand edge of the figure?	10.	No, one point does not touch the right hand edge of the figure.
11.	Is it an even numbered area to the right of the double line and below the dotted line?	11.	No, it is not an even numbered area to the right of the double line and below the dotted line.
12.	Is it an even numbered area to the left of the double line and below the dotted line?	12.	No, it is not an even numbered area to the left of the double line and below the dotted line.
13.	Is it an odd numbered area to the right of the double line and above the dotted line?	13.	No, it is not an odd numbered area to the right of the double line and above the dotted line.
14.	Is it an odd numbered area to the left of the double line and above the dotted line?	14.	No, it is not an odd numbered area to the left of the double line and above the dotted line.

Solution: The pre-selected area is #10.

Instructions for Problem 22

Pretend that lutts and mipps represent a new way of measuring distance, and that dars, wors, and mirs represent a new way of measuring time.

A man drives from Town A to Town D. He drives through Town B and Town C in order to reach Town D. Your task is to determine how many wors the entire trip took.

Remember, the words for different distances are mipps and lutts, while the words for different times are mirs, wors, and dars.

Read over the questions in the folder, and decide which one you would like to have answered first. Tell the examiner its number, and read the answer on the back of the card. Do not select more questions than you need to answer the question, but don't hesitate to select any question that you feel you need.

Questions Employed in Problem 22
And Corresponding Answers

Question	Answer
1. How far is it from A to B?	1. It is 4 lutts from A to B.
2. How far is it from B to C?	2. It is 8 lutts from B to C.
3. How far is it from C to D?	3. It is 10 lutts from C to D.
4. What is a lutt?	4. A lutt is 10 mipps.
5. What is a mipp?	5. A mipp is a way of measuring distance.
6. How many mipps are there in a mile?	6. There are 2 mipps in a mile.
7. What is a dar?	7. A dar is 10 wors.
8. What is a wor?	8. A wor is 5 mirs.
9. What is a mir?	9. A mir is a way of measuring time.
10. How many mirs are there in an hour?	10. There are 2 mirs in an hour.
11. How fast does the man drive from A to B?	11. The man drives from A to B at the rate of 24 lutts per wor.
12. How fast does the man drive from B to C?	12. The man drives from B to C at the rate of 30 lutts per wor.
13. How fast does the man drive from C to D?	13. The man drives from C to D at the rate of 30 lutts per wor.

Solution: $\frac{23}{30}$ wors

Appendix I

Instructions for Problem 23

In this problem, Bill Brown is a college student who has a summer job. The company he works for pays some employees $45. per week, some $65. per week, and some $90. per week. You are to find out how much Bill Brown earns.

You can find out the solution to the problem by selecting questions to be answered. Read over the questions, and then decide which one you would like to have answered first. Take the card from the folder, and read the answer to the question on the reverse side. Then decide on the next question you would like to have answered. Proceed in this way until you have the solution to the problem. Select as many questions as you consider necessary and sufficient to solve the problem. Do not ask more questions than you need, but do not hesitate to use any question that you feel you need. Remember, you are to find out whether Bill Brown has a $45. per week job, a $65. per week job, or a $90 per week job.

Appendix I

Questions Employed in Problem 23
And Corresponding Answers

Question	Answer
1. Who is Bill Brown?	1. Bill Brown is a college student wit a summer job. He earns either $45, $65, or $90 per week.
2.. Does Bill do manual labor?	2. Yes, Bill does do manual labor.
3. How old is Bill?	3. Bill is 18 years old.
4. Does his job require much experience?	4. No, his job does not require much experience.
5. Does Bill do outside work?	5. Yes, Bill does do outside work.
6. Does Bill work on a truck?	6. No, Bill does not work on a truck.
7. Do manual laborers earn $45 per week?	7. Manual laborers who work on trucks earn $45 per week.
8. Do 18 year old men generally earn $45 per week?	8. No, 18 year old men do not earn a $45 salary.
9. Do workers with experience earn $45 per week?	9. No, workers with experience do not earn $45 per week.
10. Do manual laborers earn $65 per week?	10. Yes, manual laborers earn $65 per week, providing they do outside lab
11. Do 18 year old men generally earn $65 per week?	11. Yes, 18 year old men do earn $65 per week, providing they are doing outside work.
12. What sort of a job is one that pays $45 per week?	12. $45 per week is just one of the different salaries that this particular company pays some of its workers.
13. What sort of job is one that pays $65 per week?	13. $65 per week is just one of the different salaries that this particular company pays some of its workers.
14. What sort of job is one that pays $90 per week?	14. $90 per week is just one of the different salaries that this particular company pays some of its workers.
15. Do manual laborers earn $90 per week?	15. No, manual laborers do not earn $90 per week.
16. Do 18 year old men earn $90 per week?	16. No, 18 year old men do not earn $90 per week.
17. Do experienced workers earn $90 per week?	17. Yes, experienced workers do earn $90 per week.
18. Do workers who earn $45 per week ever get any overtime?	18. Yes, workers who earn $45 per week sometimes get overtime.
19. Do workers who earn $65 per week ever get any overtime?	19. No, workers who earn $65 per week do not get any overtime.
20. Do workers who earn $90 per week ever get any overtime?	20. Yes, workers who earn $90 per week do sometimes get overtime, unless they are 18 years of age. These sometimes have to work late.

Solution: $65 a week is Bill Brown's salary.

Appendix I

Instructions for Problem 24

The letter A in this problem stands for an object. You are to find out whether A is an F,G, or H. You can solve the problem using only these symbols. Just as an object may have one of many characteristics, such as being red, green, or blue, or being long, medium or short, so A in this problem may be F,G, or H. It is either F,G, or H; it cannot be more than one of them. You can answer the problem if you know enough about A. You can find out about A by selecting questions to be answered. Read over the questions, and then decide which one you would like to have answered first. Take the card from the folder, and read the answer to the question on the reverse side. Then decide on the next question you would like to have answered. Proceed in this way until you have the answer to the problem.

Select as many questions as you consider necessary and sufficient to solve the problem. Do not ask more questions than you need, but do not hesitate to use any question that you feel you need.

Questions Employed in Problem 24
And Corresponding Answers

Question	Answer
1. What is A?	1. A is simply a symbol, which may also be an F, G, or H.
2. Is A also a B?	2. Yes, A is also a B.
3. Is A also a C?	3. Yes, A is also a C.
4. Is A also a D?	4. No, A is not also a D.
5. Is A also an X?	5. Yes, A is also an X.
6. Is A also a Y?	6. No, A is not a Y.
7. Is an F a B?	7. Yes, an F is a B, providing that it is also a Y.
8. Is an F a C?	8. No, an F is not a C.
9. Is an F a D?	9. No, an F is not a D.
10. Is a G a B?	10. Yes, a G is a B, providing that it is also an X.
11. Is a G a C?	11. Yes, a G is a C, providing that it is also an X.
12. What is F?	12. F is simply a symbol, that may or may not also be A.
13. What is G?	13. G is simply a symbol, that may or may not also be A.
14. What is H?	14. H is simply a symbol, that may or may not also be A.
15. Is an H a B?	15. No, an H is not a B.
16. Is an H a C?	16. No, an H is not a C.
17. Is an H a D?	17. Yes, an H is a D.
18. Is an F an L?	18. Yes, an F is an L.
19. Is a G an L?	19. No, a G is not an L.
20. Is an H an L?	20. Yes, an H is an L, unless it is also C.

Solution: A is a G.

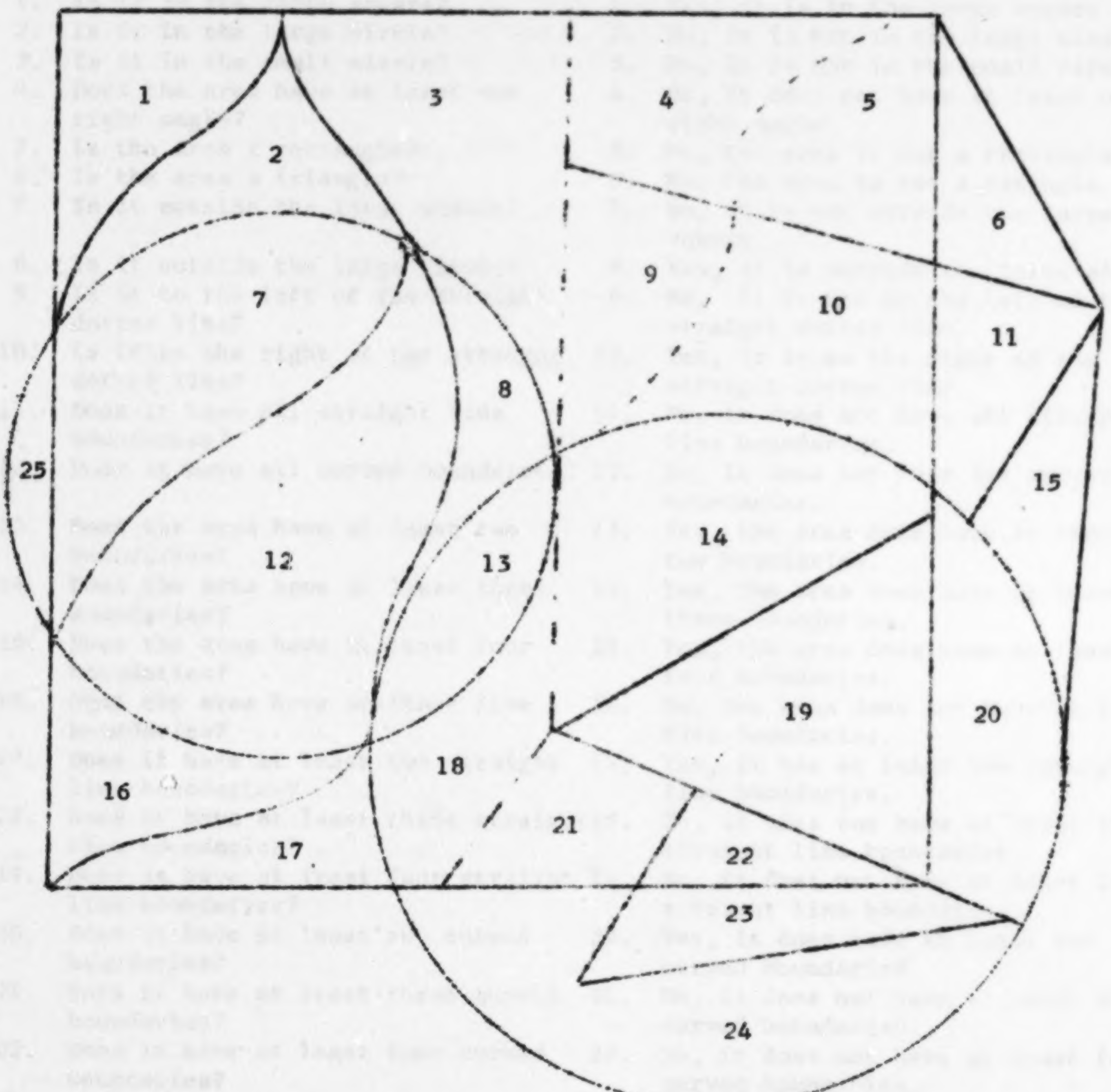
Instructions for Problem 25

The figure here is composed of overlapping geometric figures and lines, forming twenty-five areas. You will notice that an area is any enclosed part of the figure that does not have a line through it. One of the areas has been picked at random. Your task is to find out which one it was. To find the particular area, you must discover enough facts about it so that it can be distinguished from other areas. You may discover these facts by using any of the questions you like.

Proceed by reading over all the questions. Then decide on the first question you would like to have answered, and write its number on the page provided. Then take the card from the folder, and read the answer on the back of the card. After having read the answer, decide on the next question you would like to have answered. Write down its number, and take the card from the folder. When you are satisfied that you have arrived at the answer, stop drawing cards, and write down your answer. Remember, you may use as many of the cards as you need to find the correct area, but don't draw any more than you need.

Appendix I

Problem 25


1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25

Appendix I

Questions Employed in Problem 25
And Corresponding Answers

Question	Answer
1. Is it in the large square?	1. Yes, it is in the large square.
2. Is it in the large circle?	2. No, it is not in the large circle.
3. Is it in the small circle?	3. No, it is not in the small circle.
4. Does the area have at least one right angle?	4. No, it does not have at least one right angle.
5. Is the area a rectangle?	5. No, the area is not a rectangle.
6. Is the area a triangle?	6. No, the area is not a triangle.
7. Is it outside the large square?	7. No, it is not outside the large square.
8. Is it outside the large circle?	8. Yes, it is outside the large circle.
9. Is it to the left of the straight dotted line?	9. No, it is not to the left of the straight dotted line.
10. Is it to the right of the straight dotted line?	10. Yes, it is to the right of the straight dotted line.
11. Does it have all straight line boundaries?	11. No, it does not have all straight line boundaries.
12. Does it have all curved boundaries?	12. No, it does not have all curved boundaries.
13. Does the area have at least two boundaries?	13. Yes, the area does have at least two boundaries.
14. Does the area have at least three boundaries?	14. Yes, the area does have at least three boundaries.
15. Does the area have at least four boundaries?	15. Yes, the area does have at least four boundaries.
16. Does the area have at least five boundaries?	16. No, the area does not have at least five boundaries.
17. Does it have at least two straight line boundaries?	17. Yes, it has at least two straight line boundaries.
18. Does it have at least three straight line boundaries?	18. No, it does not have at least three straight line boundaries.
19. Does it have at least four straight line boundaries?	19. No, it does not have at least four straight line boundaries.
20. Does it have at least two curved boundaries?	20. Yes, it does have at least two curved boundaries.
21. Does it have at least three curved boundaries?	21. No, it does not have at least three curved boundaries.
22. Does it have at least four curved boundaries?	22. No, it does not have at least four curved boundaries.
23. Does it have a dotted boundary?	23. Yes, it does have a dotted boundary.
24. Does it have two dotted boundaries?	24. No, it does not have two dotted boundaries.
25. Does it have a straight line boundary?	25. Yes, it does have a straight line boundary.
26. Does it have a curved boundary?	26. Yes, it does have a curved boundary.

Solution: The pre-selected area is #10.

Appendix I

Instructions for Problem 26

There are objects called Ks. Your task is to find out how many of the Ks are also Ms. K might be a shorthand way of saying college student, and M a shorthand way of saying Cub fan. Then the problem would be to find out how many college students are Cub fans. Or K might stand for Democrat, and M stand for red haired. Then the problem would be to find out how many Democrats are red haired.

Begin by reading over all the questions. Decide on the first question you would like to have answered, and write its number on the page provided. Then take the card from the folder, and read the answer on the back. Then decide on the next question you would like to have answered, write down its number and take the card from the folder. When you are satisfied that you have arrived at the answer, stop taking cards and write down your answer. Remember, you may use as many cards as you need to find the answer, but don't draw more cards than you need.

Questions Employed in Problem 26 And Corresponding Answers

Question	Answer
1. How many kinds of Ks are there?	1. There are three kinds: P, B, and L.
2. How many of the Ks are Ps?	2. One half of the Ks are Ps.
3. How many of the Ks are Bs?	3. One fourth of the Ks are Bs.
4. How many of the Ks are Ls?	4. One fourth of the Ks are Ls.
5. How many of the Ks are Ds?	5. None of the Ks are Ds.
6. Is a P also an M?	6. A P is an M only when it is not a K. Ks of the P kind cannot be M.
7. How many Bs are Ms?	7. All Bs that are also Cs are Ms.
8. Are Ks of the B kind also Cs?	8. No, Ks of the B kind are not Cs.
9. How many Ls are M?	9. None of the Ls are M.
10. May a K be both a P and a B?	10. No, a K cannot be both a P and a B.
11. May a K be both a P and an L?	11. No, a K cannot be both a P and an L.
12. May a K be both a B and an L?	12. No, a K cannot be both a B and an L.
13. How many Ds are M?	13. All Ds that are not Cs are M.
14. Are Ks of the D kind also Cs?	14. There are no Ks of the D kind.

Solution: No Ks are Ms.

	Experimental	Control
N	25	25
Mean	128.00	127.36
SD	6.55	6.51

Table 1. Means and Standard Deviations of Henman-Nelson IQ for Experimental and Control Groups.

	Number of Cards		Number of Subjects Completing	Number of Correct Solutions
Problem	M	SD		
3	4.000	1.356	25	21
4	13.166	3.184	24	0
5	10.750	4.294	24	7
6	4.714	1.385	7	3
7	5.640	1.741	25	25
8	6.520	2.229	25	20
9	8.040	2.919	25	20
10	10.533	4.303	15	9
11	7.666	3.145	6	6
12	9.040	3.757	25	22
13	6.086	2.320	23	22
14	7.285	2.797	21	20
15	7.062	2.749	16	14
16	7.888	2.767	9	7
17	8.208	2.254	24	20
18	8.800	2.757	25	17
19	9.400	1.934	20	17
20	10.583	2.308	24	14
21	9.238	2.180	21	12
22	9.812	2.068	16	6
23	5.880	2.026	25	24
24	7.880	2.747	25	23

Table 2. Mean and Standard Deviation of Number of Questions, Number of Subjects Completing, and Number of Subjects Obtaining Correct Solution for each of Twenty-two Training Problems.

Appendix II

	Problems		
	7	8	9
8	-.10		
9	.03	.22	
10	.19	-.20	-.34

Session III

	Problems		
	12	13	14
13	.76**		
14	.75**	.75**	
15	.02	.44*	.39

Session IV

	Problems
	17
18	.68**

Session V

	Problem
	23
24	.25

Session VII

Table 3. Correlations Between Number of Questions Employed in Similar Problems within Same Session for Problems Completed by Fifteen or more Subjects.

**P $<$ .01.

	Group						
	Experimental			Control			Difference in Mean Change
	Midyear	Final	Change	Midyear	Final	Change	
English	84.80	82.60	-2.20	84.08	81.24	-2.84	.64
Latin	81.32	80.04	-1.28	80.60	78.56	-2.04	.76
Mathematics	84.60	84.48	- .12	83.80	80.64	-3.16	3.04*
Average[1]	83.98	83.50	- .48	83.16	81.65	-1.51	1.03

Table 4. Means of Three Academic Subjects and Overall Average of Midyear and Final Grades, Mean Changes from Midyear to Final Grades, and Differences in Changes for Experimental and Control Groups.

[1] The average was based on four academic subjects beyond those listed here.

*p $<.05$

		Groups						
		Experimental			Control			
		^1N	M	SD	^1N	M	SD	D_M
Pre-Training Problems	1	25	3.30	1.27	25	3.26	1.33	.04
	2	25	7.39	1.90	25	7.57	2.16	-.18
Post-Training Problems	25	25	6.36	1.74	25	7.96	2.29	-1.60**
	26	25	8.52	2.10	25	8.52	2.47	.00

Table 5. Mean, Standard Deviation, and Mean Difference in Number of Cards Selected in Pre-Training and Post-Training Problems by Experimental and Control Groups.

[1] Number of Subjects Attempting Problem.

$^{**}p < .01$

Appendix II

		Group			
		Experimental		Control	
		[1]Number Completing	Number of Correct Solutions	[1]Number Completing	Number of Correct Solutions
Pre-Training Problems	1	18	14	18	12
	2	18	9	18	15
Post-Training Problems	25	25	19	25	21
	26	25	14	25	9

Table 6. Number of Subjects Completing Each Pre-Training and Post-Training Problem and Number Obtaining Correct Solution.

[1] Number of Subjects Who Reached a Solution.

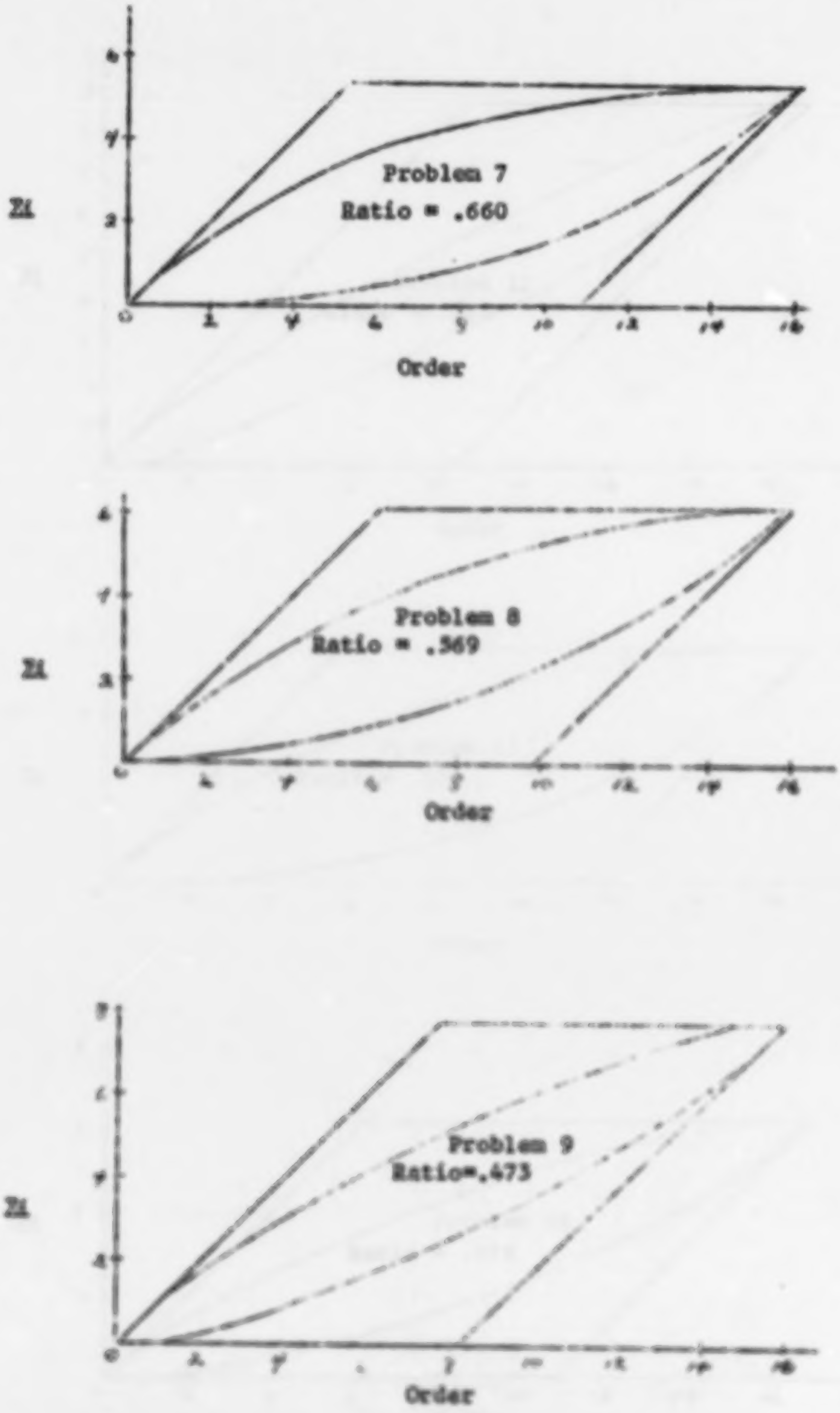


Figure 1 . Ellipsoids and Corresponding Parallelograms Generated by Performances of Experimental Subjects on Problems 7, 8, and 9 and Values of Ratios.

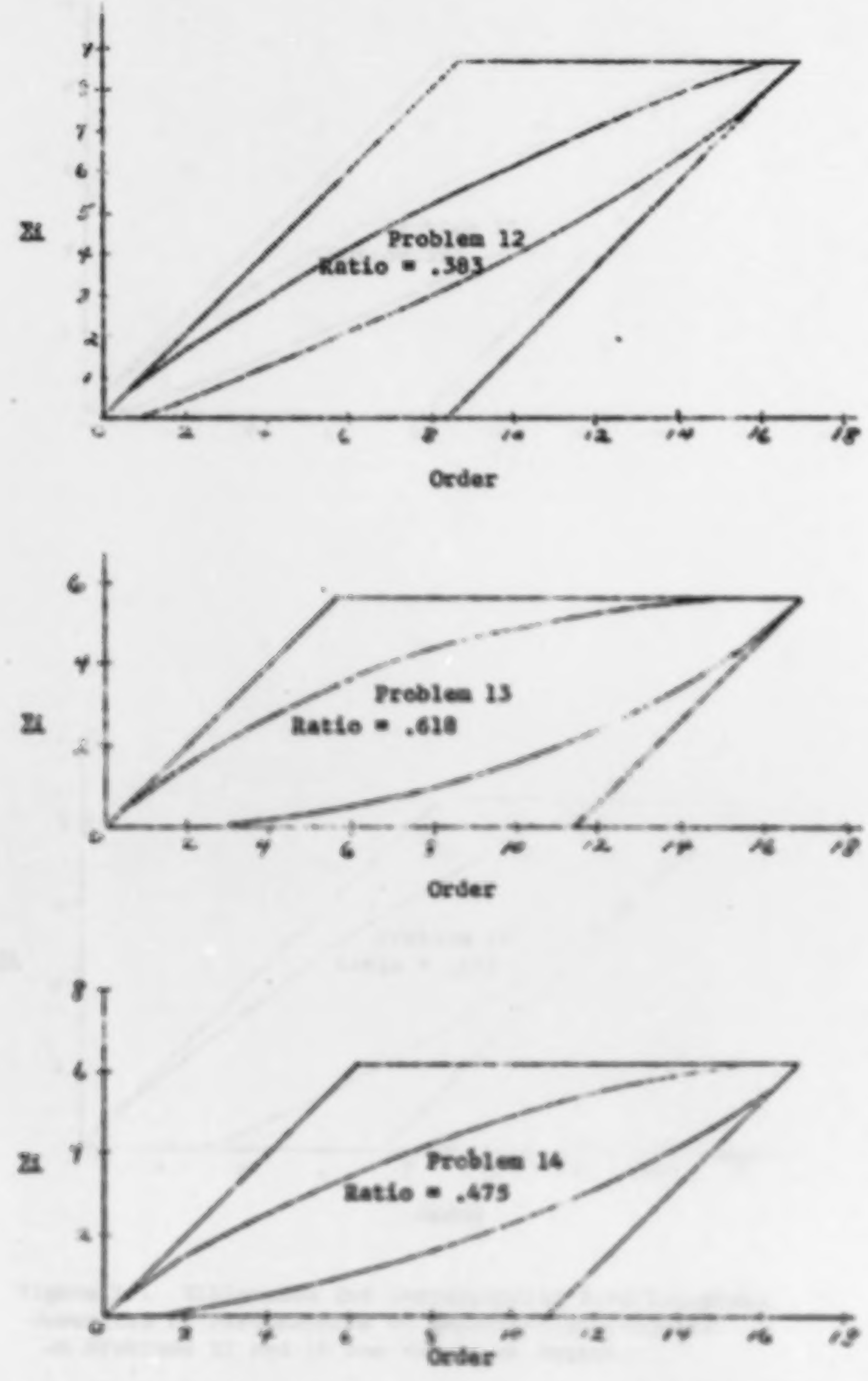


Figure 2 . Ellipsoids and Corresponding Parallelograms Generated by Performances of Experimental Subjects on Problems 12, 13, 14 and Values of Ratios.

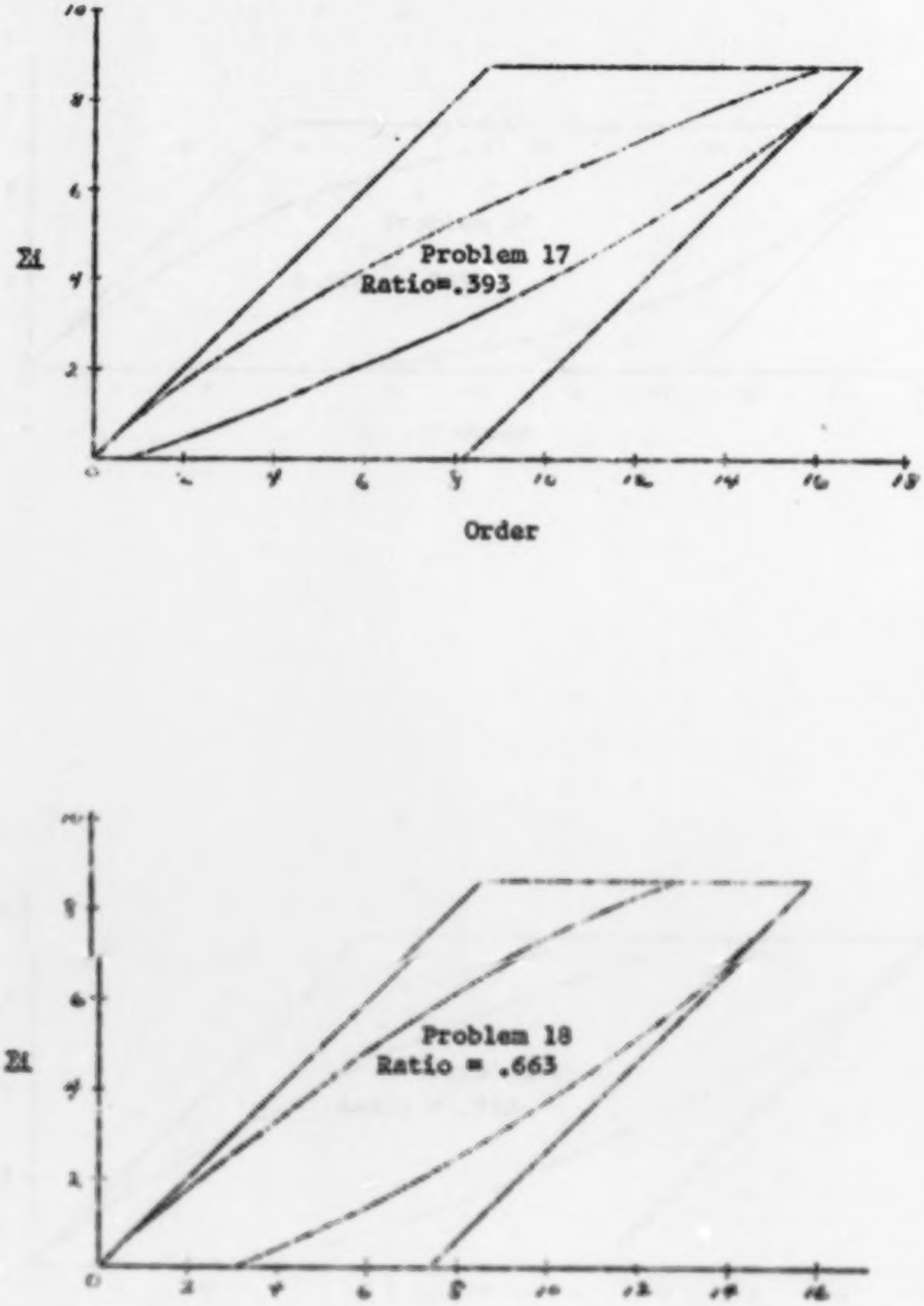


Figure 3 . Ellipsoids and Corresponding Parallelograms Generated by Performances of Experimental Subjects on Problems 17 and 18 and Values of Ratios.

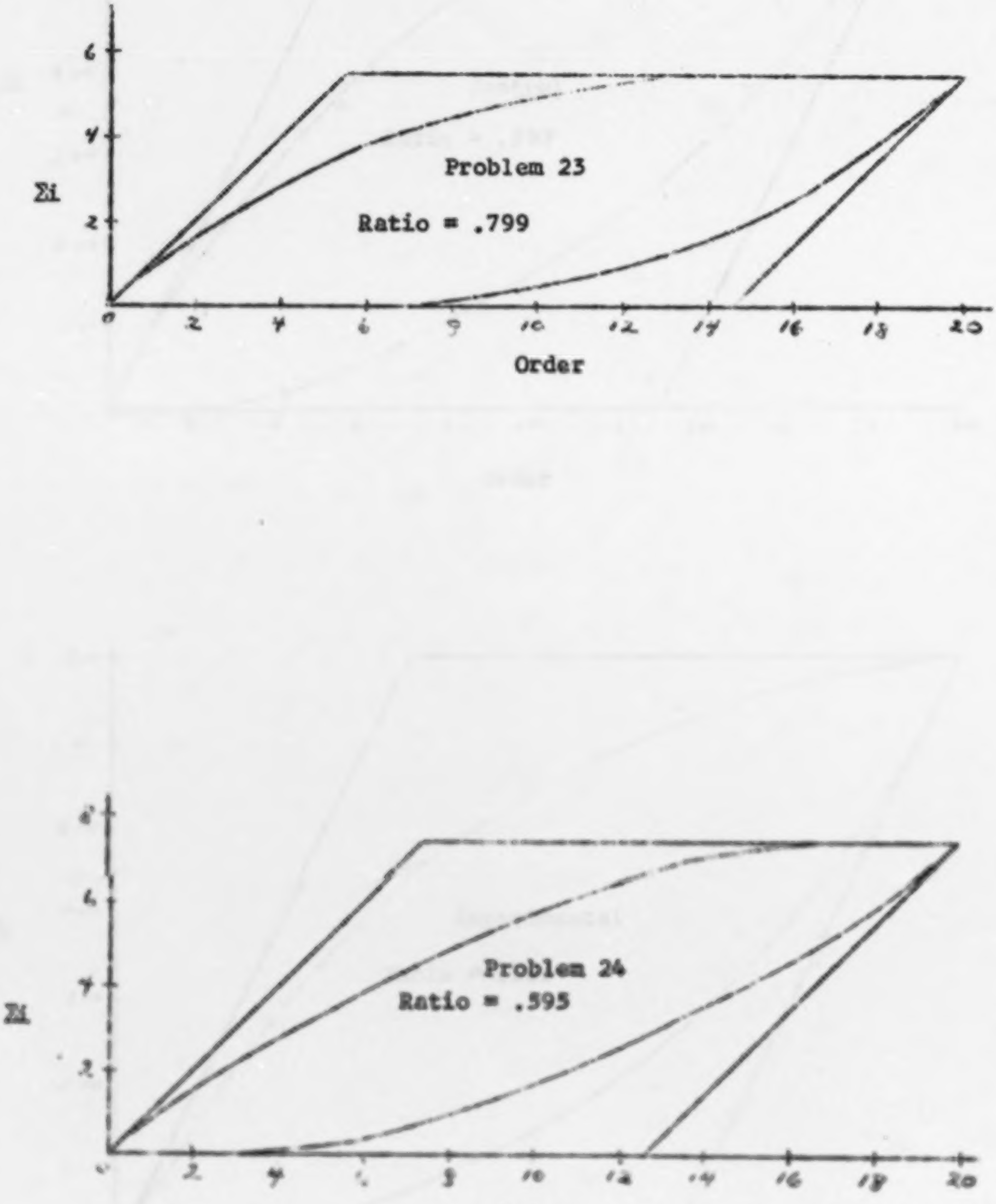


Figure 4 . Ellipsoids and Corresponding Parallelograms Generated by Performances of Experimental Subjects on Problems 23 and 24 and Values of Ratios.

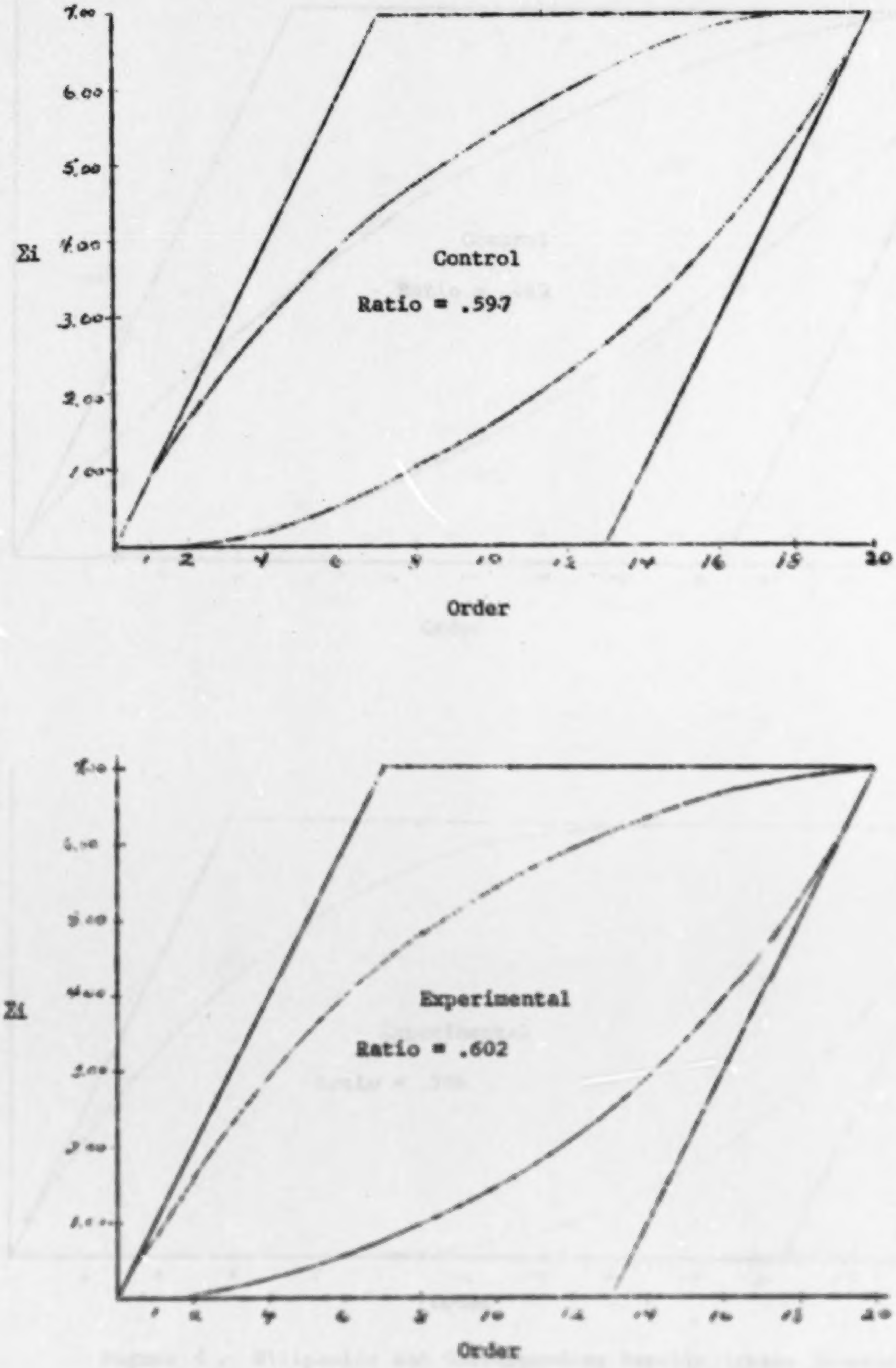


Figure 5 . Ellipsoids and Corresponding Parallelograms Generated by Performances of Experimental and Control Subjects on Pre-Training Problem 2 and Values of Ratios.

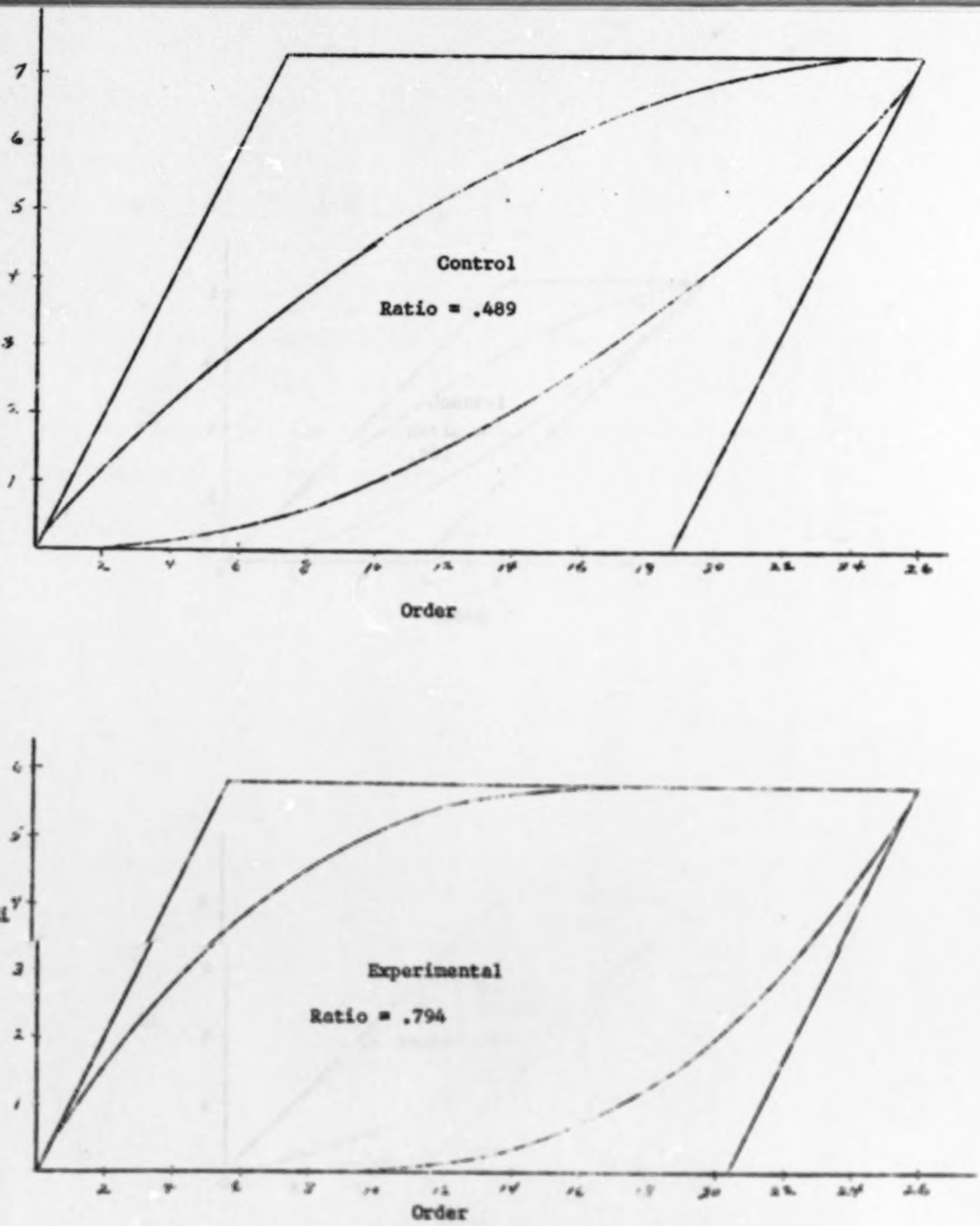


Figure 6. Ellipsoids and Corresponding Parallelograms Generated by Performances of Experimental Subjects and Control Subjects on Post-Training Problem 25 and Values of Ratios.

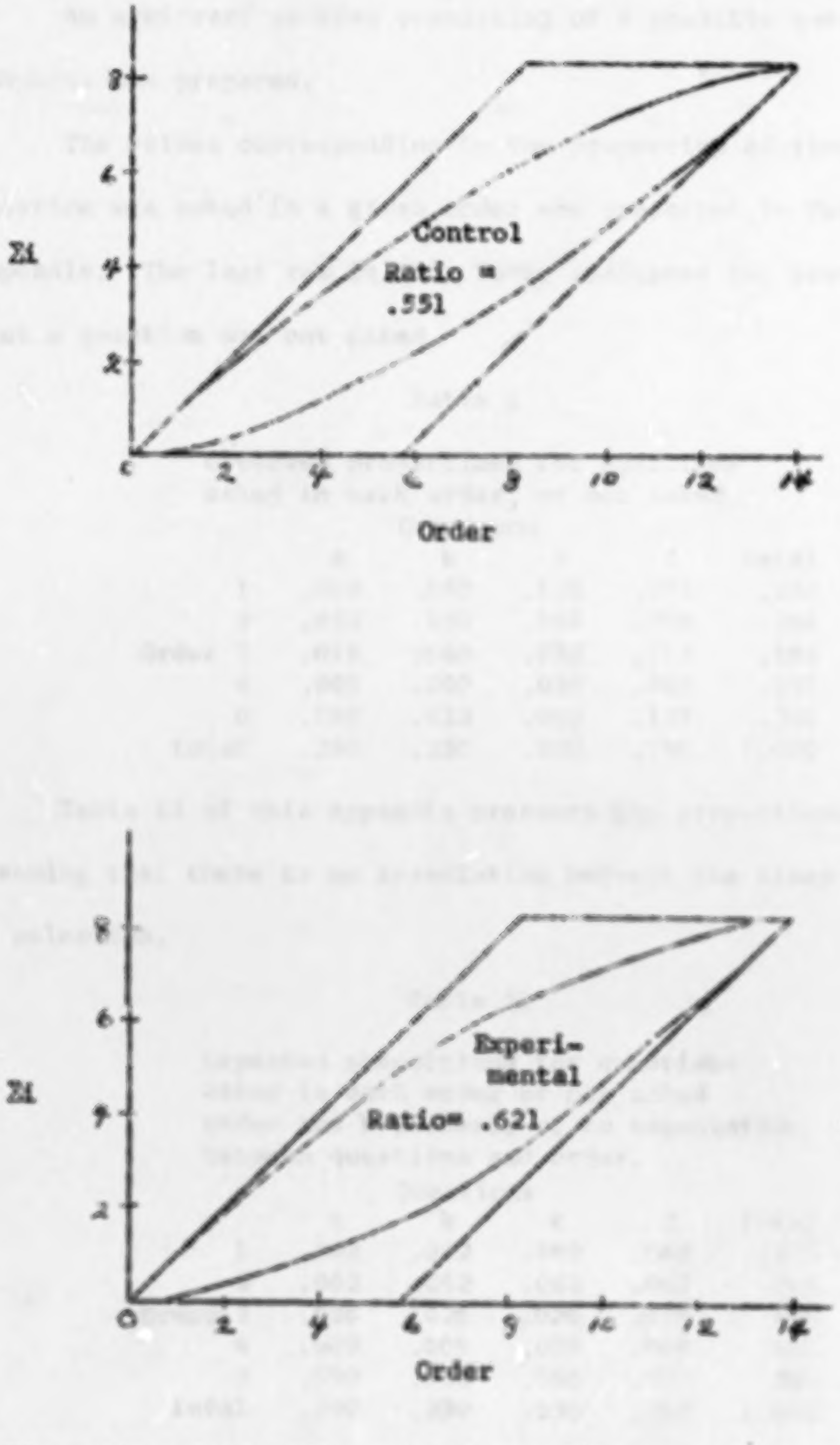


Figure 7 . Ellipsoids and Corresponding Parallelograms Generated by Performances of Experimental Subjects and Control Subjects on Post-Training Problem 26 and Values of Ratios.

Appendix IV

1.

Demonstration of Sequential Evaluation of Problem Solving Processes

An arbitrary problem consisting of 4 possible questions and 100 subjects was prepared.

The values corresponding to the proportion of times that each question was asked in a given order are presented in Table i of this Appendix. The last row in this Table indicates the proportion of times that a question was not asked.

Table i

Observed proportions for questions asked in each order, or not asked.

		Questions				
		a	b	c	d	total
	1	.010	.040	.125	.075	.250
	2	.032	.130	.068	.018	.248
Order	3	.018	.040	.032	.015	.105
	4	.005	.007	.020	.005	.037
	0	.185	.033	.005	.137	.360
	total	.250	.250	.250	.250	1.000

Table ii of this Appendix presents the proportions expected assuming that there is no association between the items and the order of selection.

Table ii

Expected proportions for questions asked in each order or not asked under the hypothesis of no association between questions and order.

		Questions				
		a	b	c	d	total
	1	.062	.062	.062	.062	.250
	2	.062	.062	.062	.062	.248
Order	3	.026	.026	.026	.026	.105
	4	.009	.009	.009	.009	.037
	0	.090	.090	.090	.090	.360
	total	.250	.250	.250	.250	1.000

Appendix IV

The number of possible tactics is 64, of which 4 tactics will consist of only one question, 12 of two questions, 24 tactics for sequences of 3 questions and 24 tactics for sequences of 4 questions.

Take tactics formed by the same three questions asked in different orders. Let the questions be: a b c.

Cumulating observed values and observed minus expected value, for both questions asked and not asked in all tactics having questions a b c we obtain:

Tactic	Sum Total (observed)	Sum Total (observed - expected)	Sum Total Non Selected (observed)	Sum Total Non Selected (observed - expected)
cba	.273	.123	.137	.047
cab	.197	.047	.137	.047
bac	.104	-.046	.137	.047
bca	.126	-.024	.137	.047
abc	.172	.022	.137	.047
acb	.118	-.032	.137	.047

Graphing tactics cba, abc and bac gives Figure 1 of this Appendix. It can be noticed that both the observed and observed minus expected curves differ for the three cases under consideration.

The convex set of Figure 2 of this Appendix was obtained by first plotting on the abscissa the sum total of the observed values for all the tactics and on the ordinate the sum total of the observed minus expected values and then tracing the boundaries.

The numbers written on the edge of the limits of the convex set refer to tactics. Those falling within the limits are represented by dots. Notice that 2 different tactics may have the same sum total for observed values but different values for the sum total of the observed minus expected.

Appendix IV

It is intriguing that the tactics limiting the boundaries seem to follow a certain pattern. In Table iii we present the composition of these tactics.

Table iii

Tactics that define the limits of performance.

Number of Tactics	Components
33	c
34	cb
34	cba
36	cbad
25	bcad
16	adcb
12	ad
1	a
17	b
49	d

For the nonselected questions the convex set given in Figure 3 of this Appendix was obtained. (The numbers in this figure do not correspond to the numbers in Figure 2). Notice that tactics 23 and 60 in Figure 2 are close together but that in terms of questions not asked they correspond to Point 10 and 4 respectively (Fig. 3), which are set wide apart.

Appendix IV

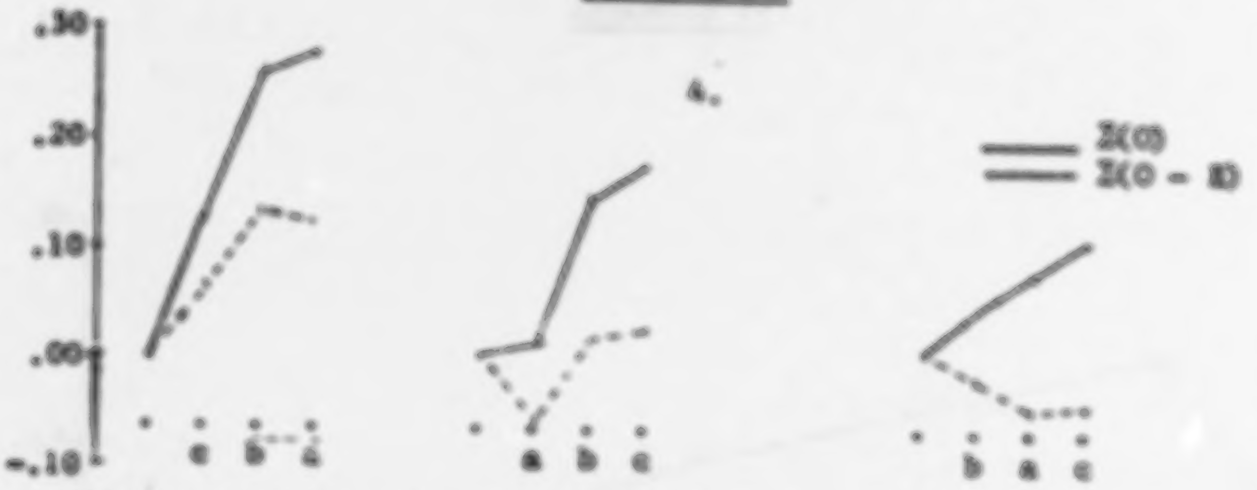


Figure 1. Observed and observed minus expected performance curves for three cases.

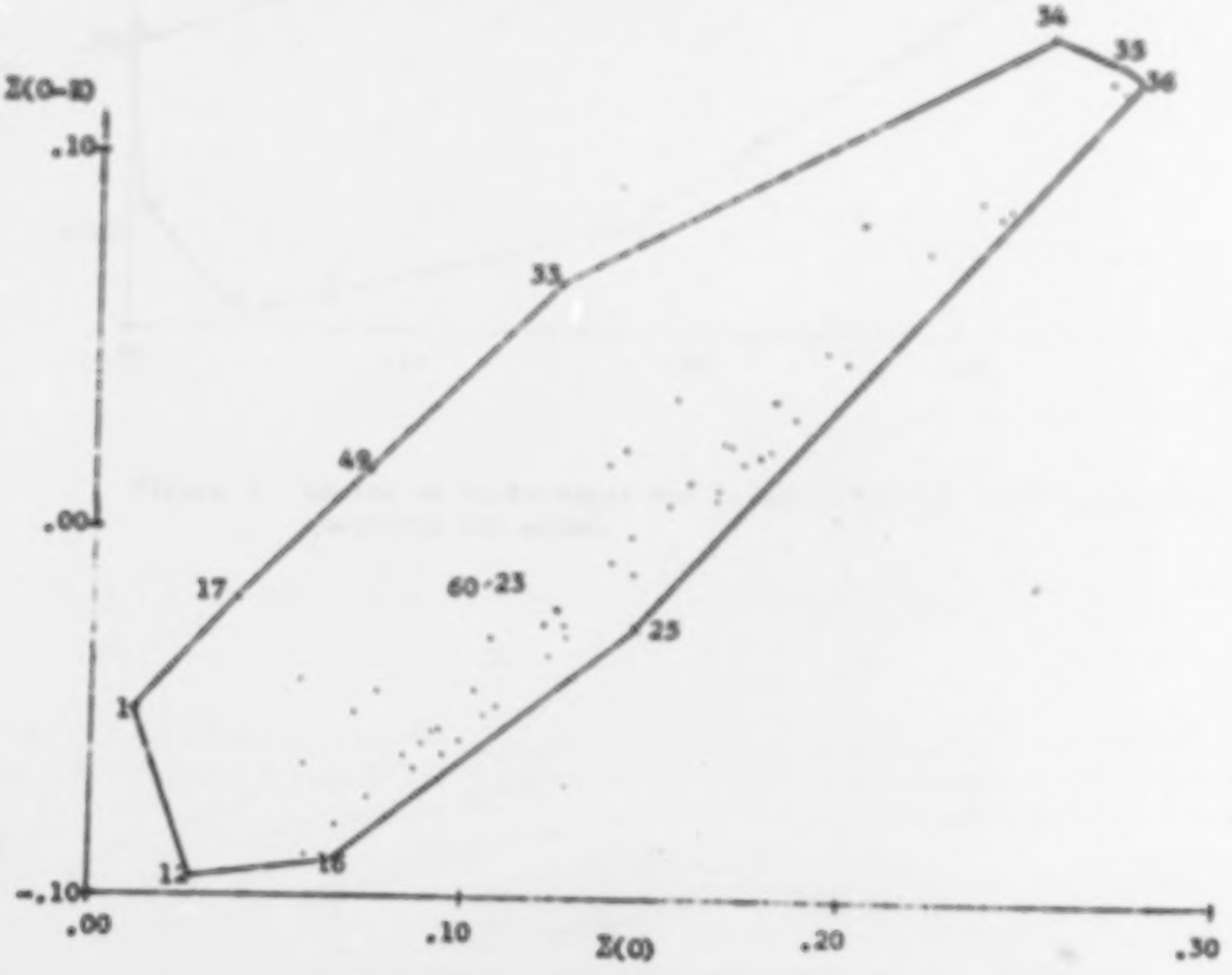


Figure 2. Limits of Performance for the hypothetical test under the hypothesis of no association between questions and order of selection.

Appendix IV

3.

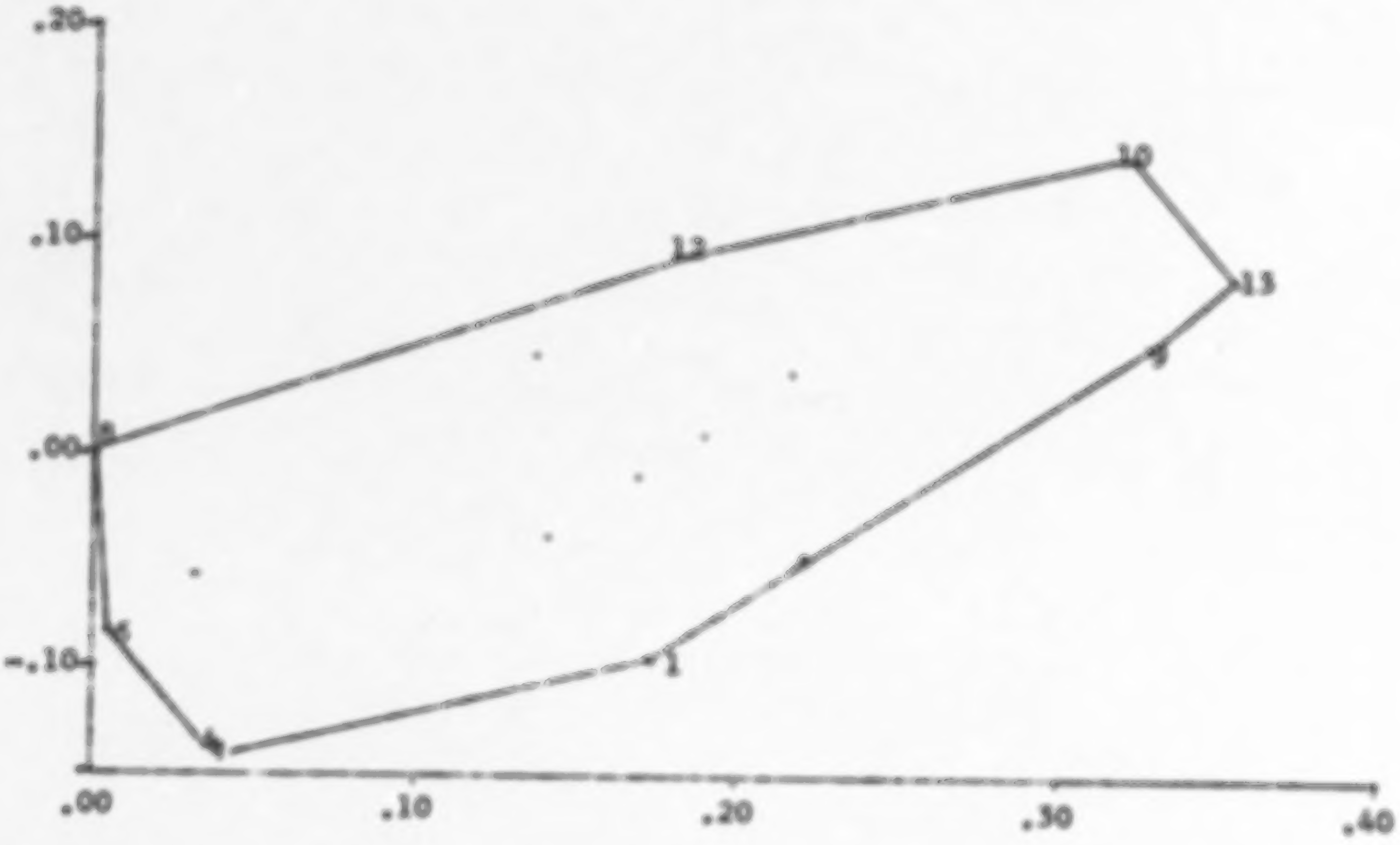


Figure 3. Limits of Performance for the hypothetical test considering questions not asked.